Math 214 Spring 2006 **SHOW ALL YOUR WORK** Quiz **Five**

1. TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) **TRUE or FALSE?** "A 4×4 matrix with a row of zeros is not invertible."

TRUE

A 4x4 matrix with zero row will have a rref(A) with a zero row. Thus the rank of this 4x4 matrix will be less than 4. Only a 4x4 matrix with rank=4 is invertible. Thus this matrix is ~~invertible~~ NOT INVERTIBLE.

(b) **TRUE or FALSE?** "A matrix with 1's down the main diagonal is invertible."

FALSE

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ has 1's down the main diagonal but is NOT INVERTIBLE since $\det\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0$.

Also $\text{rref}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \neq I$, so $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ is NOT INVERTIBLE

(c) **TRUE or FALSE?** "If A is invertible, then A^{-1} is invertible."

TRUE

If A^{-1} exists then its inverse ♡ has the property $♡ \cdot A^{-1} = I$ and $A^{-1} \cdot ♡ = I$

Since we know A^{-1} exists we know

$A \cdot A^{-1} = I$ and $A \cdot A^{-1} = I$

thus ♡, which is the inverse of A^{-1}, is A

$A = ♡ = (A^{-1})^{-1}$