

Math 214, Fall 2003 Final Exam, Question 8.

Given $A = \begin{bmatrix} 1 & 5 & 3 & 1 & 0 \\ -1 & -3 & 0 & 0 & 2 \\ 3 & -3 & 1 & -6 & 1 \\ 2 & -4 & -1 & -5 & 0 \end{bmatrix}$ with $\text{rref}(A) = R = \begin{bmatrix} 1 & 0 & 0 & -1.5 & -0.5 \\ 0 & 1 & 0 & 0.5 & -0.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Fill in the blanks.

a. The rank of the matrix A is 3

b. $\text{null}(A)$ is a subspace of \mathbb{R}^5

c. The dimension of $\text{col}(A)$ is 3

d. How many vectors are there in a basis of $\text{row}(A)$? 3

e. $\text{row}(A)$ is a subspace of \mathbb{R}^5 .

f. $\text{null}(A)$ is spanned by the vectors $\begin{pmatrix} 1.5 \\ -0.5 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

g. The span of the columns of R is all of \mathbb{R}^3

TRUE or **FALSE** (circle one).

It is a 3-D object (subspace) in \mathbb{R}^4 , This is NOT the same thing as "all of \mathbb{R}^3 "

h. $A\vec{x} = \vec{b}$ will be solvable for any $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix}$.

TRUE or FALSE (circle one).

This describes $\text{col}(A)$, $\vec{b} \in \text{col}(A) \Rightarrow A\vec{x} = \vec{b}$ has unique soln

i. An example of a basis for $\text{col}(A)$ is $\begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ -3 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ -1 \end{pmatrix}$