Test 1: Linear Systems

Math 2	214 Sp	ring	2007
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Friday March 2 2:30pm-3:25pm

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Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is a 55-minute, no-notes, closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

Pledge: I,	pledge	my
honor as a human being and Occidental student, that	I have	fol-
lowed all the rules above to the letter and in spirit.		

No.	Score	Maximum		
1		20		
2		25		
3		30		
4		25		
BONUS		10		
Total		100		

1. Span, Linear Independence, Rank. 20 points.

Consider the matrix $A = \begin{bmatrix} 3 & 6 \\ 9 & 18 \end{bmatrix}$.

(a) (4 points.) Find the reduced row echelon form of A, rref(A).

(b) (4 points.) With or without your knowledge of rref(A), what is the rank of the matrix A? **EXPLAIN YOUR ANSWER.**

(c) (4 points.) With or without your knowledge of rref(A), what is the span of the columns of matrix A? **EXPLAIN YOUR ANSWER.**

(d) (4 points.) With or without your knowledge of rref(A), discuss the linear independence of the columns of the matrix A. **EXPLAIN YOUR ANSWER.**

(e) (4 points.) With or without your knowledge of rref(A), discuss whether the matrix A^{-1} exists. **EXPLAIN YOUR ANSWER.**

2. Row reduction, Reduced Row Echelon Form, Identity, Invertibility. 25 points.

(a) (8 points.) Show that
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & -1 \\ 2 & 2 & 6 \end{bmatrix}$$
 can be transformed into $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ using elementary row operations

(b) (8 points.) Show that
$$I_3$$
 can be transformed into $B = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ using elementary row operations.

(c) (9 points.) Is it possible to transform A into B using elementary row operations? Explain why you can not or explain why you can, without going through the actual work of proving the result. Is A invertible? Is B invertible? How do you know? EXPLAIN YOUR ANSWERS.

3. Matrix Operations, Trace, Transpose. 30 points.

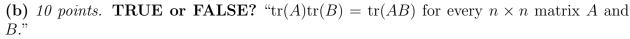
TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

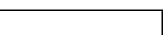
The **trace** of a $n \times n$ matrix A (where A_{ij} is the element in the i^{th} row and j^{th} column) is defined as **the sum of the diagonal elements of A** and denoted tr(A). In other words,

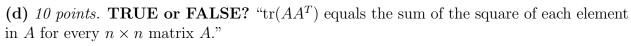
$$\operatorname{tr}(A) = \sum_{i=1}^{n} A_{ii}$$

(a) 10 points. TRUE or FALSE? "The trace of the $n \times n$ identity matrix I_n , $\operatorname{tr}(I_n)$ equals n."











4.	Linear	Systems,	Equations	of Planes	and	Lines.	23	points.
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Consider the linear system

$$3x + 5y - 4z = 0$$
$$-3x - 2y + 4z = 0$$
$$6x + y - 8z = 0$$

(a) (10 points.) Find the non-trivial solution(s) of the linear system.

(b) (10 points.) What is the geometrical object which represents the solution of the linear system? Write down its equation in vector form.

(c) (5 points.) What is the smallest distance between the geometrical object which represents the solution of the linear system and the origin (0,0,0). EXPLAIN YOUR ANSWER.

BONUS QUESTION. Analytic Geometry, Projections. (10 points.)

Find the smallest distance between the geometrical object which represents the solution to the linear system in Question 4 and the point (1, 1, 1).