Class 11: Wednesday February 15

SUMMARY Matrix Algebraic Operations

CURRENT READING Poole 3.2

Summary

Let’s do math with matrices. Yay. We’ll summarize our knowledge of algebraic properties of matrices.

Homework Assignment

HW # 11: Section 3.2: 1,2,3,4,5,14,24,37, 44; EXTRA CREDIT # 45, 46 DUE FRI FEB 17

1. Algebraic Properties of Matrix Addition and Scalar Multiplication

Let $A,B$ and $C$ be matrices of size $m \times n$ and let $O$ be the zero matrix of size $m \times n$. Let $c$ and $d$ be scalars.

(1) $A + B = B + A$ (Commutativity of Addition)

(2) $A + O = A$ (Existence of Additive Identity)

(3) $A + (-A) = O$ (Existence of Additive Inverse)

(4) $c(A + B) = cA + cB$ (Distributivity of Scalar Multiplication)

(5) $(c + d)A = cA + dA$ (Distributivity of Scalar Addition)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

2. Algebraic Properties of Matrix Multiplication

(1) $A(BC) = (AB)C$ (Associativity of Matrix Multiplication)

(2) $A(B + C) = AB + AC$ (Distributivity of Left Matrix Multiplication)

(3) $(A + B)C = AC + BC$ (Distributivity of Right Matrix Multiplication)

(4) $k(AB) = (kA)B = A(kB)$ (Associativity of Scalar Multiplication)

(5) $I_mA = A = AI_n$ (Existence of Multiplicative Identity)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

(7) $1A = A$ (Existence of Multiplicative Identity)

Exercise

Is $(A + B)^2 = A^2 + 2AB + B^2$ for all matrices $A$ and $B$? Prove your answer!
3. Linear Independence and Span With Matrices

Recall we previously defined the concepts of linear independence and span involving vectors in \( \mathbb{R}^n \).

**GroupWork**

Write down a one sentence definition in YOUR OWN WORDS explaining linear independence and span.

**Linear Independence**

**Span**

**EXAMPLE**

Consider \( A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \), \( A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and \( A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \). Are these matrices linearly independent?

What is the span of these matrices?