Linear Systems

Math 214 Spring 2006
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Fowler 307 MWF 2:30pm - 3:25pm
http://faculty.oxy.edu/ron/math/214/06/

Class 5: Wednesday February 1

SUMMARY Understanding Linear Systems of Equations
CURRENT READING Poole 2.1

OUTLINE
Today we will discover different ways of looking at linear systems and discover an interesting fact common to all linear systems.

Homework Assignment
HW #5: Section 1.3: 7, 14, 18, 27, 29; EXTRA CREDIT 25: DUE FRI FEB 3

Consider the vectors \( \vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \) and \( \vec{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \)

One of the central ideas of this Linear Systems course concerns Linear Combinations of Vectors.

One of the basic questions is when does a linear combination of vectors equal another vector? In other words, can you find a linear combination of \( \vec{v} \) and \( \vec{w} \) such that

\[
c\vec{v} + d\vec{w} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}
\]

OR

\[
c\vec{v} + d\vec{w} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}
\]

OR

\[
c\vec{v} + d\vec{w} = \text{any 2x1 vector?}
\]

NOTE: If we take ALL linear combinations of \( \vec{v} \) and \( \vec{w} \) we can produce every vector in the entire plane.

What’s the relationship between this and solving systems of equations? We could write the problem above as a linear system

\[
\begin{align*}
3c + d &= 7 \\
c + 5d &= 7
\end{align*}
\]

(row form)

OR

\[
c \begin{bmatrix} 3 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}
\]

(column form)
GroupWork

Solve one of the following systems of equations.

System A.

\[ \begin{align*}
2x - y &= 1 \\
-4x + 2y &= 2
\end{align*} \]

System B.

\[ \begin{align*}
2x - y &= 1 \\
-4x + y &= 2
\end{align*} \]

System C.

\[ \begin{align*}
2x - y &= 1 \\
-6x + 3y &= -3
\end{align*} \]

Let’s graph each of the above systems of equations on the \( xy \)-plane below.

Q: Before doing so, what do you expect to see? What do you see?
1. Algebraic and Geometric Interpretations of Linear Systems

\[
\begin{align*}
4x - y &= 4 \\
2x - 3y &= -5
\end{align*}
\]

The above is called the **row form** of the system of equations.

Geometrically, the row form can be viewed as:

\[
x \begin{bmatrix} 4 \\ 2 \end{bmatrix} + y \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}
\]

The above is called the **column form** of the system of equations.

Geometrically, the column form can be viewed as:

\[
\begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}
\]

The above is called the **matrix form** of the system of equations.

Geometrically, the matrix form can be viewed as:
Warm-up
Consider 2 random lines in 2-dimensional space (the regular Cartesian plane). What are the possible scenarios these two random lines can produce? Draw pictures below:

Consider 2 random planes in 3-dimensional space. What are the possible scenarios these 2 random planes can produce? Write them down below.

Consider 3 random planes in 3-dimensional space. What are the possible scenarios these 3 planes can produce? Write them below.

DISCUSSION
What is the connection between the above scenarios and the question of when a linear system of equations has a unique solution? Or no solution?

**DEFINITION:** consistent
If a system of linear equations has at least one solution then it is called a consistent linear system. Otherwise, it is called an inconsistent linear system.

**DEFINITION:** singular
If a system of linear equations does not have a unique solution then it is called a singular linear system. Otherwise, it is called a non-singular linear system.