Math 214 Spring 2006 **SHOW ALL YOUR WORK** Quiz **One**

1. Consider $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} k^2 \\ k \\ -3 \end{bmatrix}$ where k is some unknown scalar.

(a) *3 points.* Find the values of the scalar k for which the two vectors $\vec{u}$ and $\vec{v}$ are **orthogonal** to each other.

$\vec{u} \cdot \vec{v} = 0$

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} K^2 \\ K \\ -3 \end{pmatrix} = 1 \cdot K^2 - 1 \cdot K + 2 \cdot (-3)$$
$$= K^2 - K - 6$$
$$= (K-3)(K+2) = 0$$
$$K = 3 \text{ or } K = -2$$

$\begin{pmatrix} 9 \\ 3 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}$ are both orthogonal to $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(b) *2 points.* Is it possible to find values of k for which the two vectors $\vec{u}$ and $\vec{v}$ are **parallel** to each other? **EXPLAIN YOUR ANSWER.**

$\vec{u} = c\vec{v}$

$$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = c \begin{pmatrix} K^2 \\ K \\ -3 \end{pmatrix} \Rightarrow \begin{matrix} -3c = 2 \\ Kc = -1 \\ K^2c = 1 \end{matrix} \quad \begin{matrix} \Rightarrow c = -2/3 \\ \Rightarrow K\cdot\frac{-2}{3} = -1 \Rightarrow K = \frac{3}{2} \end{matrix}$$

$$\left(\frac{3}{2}\right)^2 \cdot \left(\frac{-2}{3}\right) \overset{?}{=} 1$$

$$\frac{3}{2} \overset{?}{=} 1$$

No!

There exists no c and K so that $\vec{u} = c\vec{v}$, i.e. $\vec{u}$ and $\vec{v}$ are parallel.

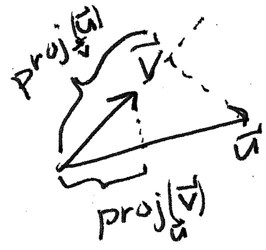
(c) *3 points.* Let $k = 0$ to produce a specific known vector $\vec{v}$. Compute $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{proj}_{\vec{u}}(\vec{v})$.

$\vec{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \vec{v} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$

$$\text{proj}_{\vec{u}}(\vec{v}) = \left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \left(\frac{-6}{1+1+4}\right)\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\text{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\frac{-6}{0+0+9}\right)\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = -\frac{2}{3}\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

(d) *2 points.* Are your answers in part (c) different? Is this a surprise? **EXPLAIN YOUR ANSWER.**



You shouldn't be surprised that the projection $\vec{u}$ onto $\vec{v}$ is NOT the same as the projection of $\vec{v}$ onto $\vec{u}$