Math 214 Spring 2006 **SHOW ALL YOUR WORK** **BONUS** Quiz 3

1. In the following two cases, determine whether span $(\vec{u}, \vec{v}, \vec{w}) = \mathbb{R}^3$ or not.

(a) *4 points.* Suppose $\vec{u} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$. Does span $(\vec{u}, \vec{v}, \vec{w}) = \mathbb{R}^3$? **EXPLAIN YOUR ANSWER!**

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow[R_2' = R_2 - R_1]{} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This means that the only solution to $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{0}$ is $c_1 = c_2 = c_3 = 0$. Thus, these 3 vectors are linearly independent and span $\mathbb{R}^3$

(b) *4 points.* Suppose $\vec{u} = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$. Does span $(\vec{u}, \vec{v}, \vec{w}) = \mathbb{R}^3$? **EXPLAIN YOUR ANSWER!**

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

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A zero row ~~means~~ means that there are solutions to

$c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = \vec{0}$

$c_1 = -c_3 \quad c_2 = -c_3 \quad$ let c_3 be 1

$$-\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The vectors are linearly dependent.

(c) *2 points.* Are the vectors in part **(a)** linearly independent or linearly dependent? Are the vectors in part **(b)** linearly independent or linearly dependent? **EXPLAIN YOUR ANSWER!**

If 3 vectors in $\mathbb{R}^3$ are linearly independent they span $\mathbb{R}^3$.

If 3 vectors in $\mathbb{R}^3$ are lin dependent then they can not span $\mathbb{R}^3$.