1. (a) Show that the plane given by \(4x - y - z = 6\) and the line given by \(x = t, y = 1 + 2t, z = 2 + 3t\) intersect.

Substitute \(x = t, y = 1 + 2t, z = 2 + 3t\) into \(4x - y - z = 6\)

\[
4(t) + (1+2t) - (2+3t) = 6 \\
4t - 1 - 2t - 2 - 3t = 6 \\
-3 - t = 6 \\
-t = 9 \\
t = -9
\]

When \(t = 9\), \(x = -9\)
\(y = 1 + 2(-9) = -17\)
\(z = 2 + 3(-9) = -25\)

\[
\begin{align*}
4(-9) - 1(-17) - (-25) &= 6 \\
-36 + 17 + 25 &= 6 \\
-36 + 42 &= 6 \\
6 &= 6
\end{align*}
\]

\(\begin{pmatrix} -9 \\ -17 \\ -25 \end{pmatrix}\) is on the line

\(\begin{pmatrix} -9 \\ -17 \\ -25 \end{pmatrix}\) is also on the plane

(b) Find the acute angle of intersection between the line and the plane given in part (a).

Given \(4x - y - z = 6\), the vector \(\begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}\) is orthogonal (or normal) to the plane.

The line \(\begin{pmatrix} t \\ 1 + 2t \\ 2 + 3t \end{pmatrix} = t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}\), thus \(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\) is the direction the line is in.

The angle between the line and plane is given by the dot product formula

\[
\cos \theta = \frac{(4, -1, -1) \cdot (1, 2, 3)}{|(4, -1, -1)| \cdot |(1, 2, 3)|} = \frac{4 - 2 - 3}{\sqrt{4^2 + 1^2 + 1^2} \cdot \sqrt{1^2 + 2^2 + 3^3}} = \frac{-1}{\sqrt{18} \cdot \sqrt{14}}
\]

\[
= -\frac{1}{3} = -\frac{1}{\sqrt{18}} \quad \theta = 93.5^\circ, \text{ Actual Angle is } 180^\circ - 93.5^\circ = 86.5^\circ.
\]