Linear Systems

Math 214 Spring 2001 — Fowler 112 MWF 1:30pm - 2:25pm

http://www.ron.oxy.edu/math/214/01/

PROPERTIES OF THE DETERMINANT (§5.1)

You should think of the determinant as a *function*. You input a matrix, and the output is a number. The first three properties below are all that is required to define (and calculate) the determinant of a square matrix A. The remaining properties follow from the first three and will make working with determinants much easier. This is an example of an *axiomatic system*. The first three *axioms* are given properties of the determinant. Everything else can be deduced from these three axioms.

1. The determinant is a linear function of the first row.

2. The determinant changes sign when two rows are exchanged.

3. The determinant of the n by n identity matrix is 1.

4. If two rows of A are equal, then $\det A = 0$.

5. Subtracting a multiple of one rows from another row leaves $\det A$ unchanged.

6. A matrix with a row of zeros has $\det A = 0$.

7. If A is a triangular matrix the $\det A$ equals the product of the main diagonal entries.

8. If A is singular then $\det A = 0$. If A is invertible then $\det A \neq 0$.

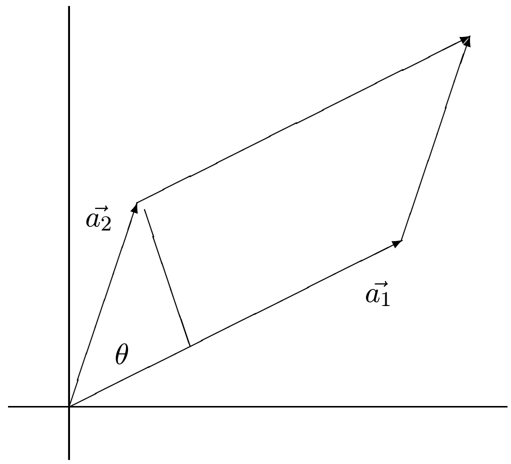
9. The determinant of AB is the product of the separate determinants: $|AB| = |A||B|$.

10. The transpose A^T has the same determinant as A.

Geometric Introduction.

For a 2×2 matrix $A = \left[\begin{array}{c} \vec{a_1} \\ \hline \vec{a_2} \end{array}\right] = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$, the *determinant* of A is the oriented area of the resulting parallelogram.

Notation: The determinant of A is denoted $\det(A)$ or $|A|$.



$$\begin{aligned}
(AREA)^2 &= \|\vec{a_1}\|^2\|\vec{a_2}\|^2 \sin^2\theta \\
&= \|\vec{a_1}\|^2\|\vec{a_2}\|^2(1-\cos^2\theta) \\
&= \|\vec{a_1}\|^2\|\vec{a_2}\|^2 - (\|\vec{a_1}\|\|\vec{a_2}\|\cos\theta)^2 \\
&= (\vec{a_1}\cdot\vec{a_1})(\vec{a_2}\cdot\vec{a_2}) - (\vec{a_1}\cdot\vec{a_2})^2 \\
&= (w^2+x^2)(y^2+z^2) - (wy+xz)^2 \\
&= (wy)^2+(wz)^2+(xy)^2+(xz)^2 - ((wy)^2+2wxyz+(xz)^2) \\
&= (wz)^2 - 2wxyz + (xy)^2 \\
&= (wz-xy)^2
\end{aligned}$$

So we see that for the 2×2 matrix A, $|A| = wz - xy$.

For a 3×3 matrix $A = \left[\begin{array}{c} \vec{a_1} \\ \hline \vec{a_2} \\ \hline \vec{a_3} \end{array}\right]$, the absolute value of the determinant of A is the volume of the resulting parallelepiped.

These ideas of areas and volumes can be found in §5.3.