Study Problems for Exam 1 in Math 212 Fall 2014

(1) If $f(\omega,x) = 2\omega x^2$, find $f(x,x^2)$.
(2) Sketch both a contour diagram and a graph of sections with $y$ fixed, for the surface $z = \sqrt{x^2 + y^2} - 1$. Then sketch the surface.
(3) Write down a function $f(x,y,z)$ for which the surface in problem (2) is a level surface. How many such functions are there?
(4) How far apart (shortest distance) are the spheres $x^2 + (y - 2)^2 + (z + 3)^2 = 1$ and $(x - 3)^2 + y^2 + (z + 2)^2 = 5$? Sketch the second sphere.
(5) Sketch the plane tangent to $f(x,y) = x^2 y + \frac{x}{y} + 1$ at (2,1).
(6) Sketch the contours of $f(x,y) = x + y^2$.
(7) Here are two points: $P=(1,2,3)$, $Q=(-2,2,4)$. Write down the coordinates of a third point $R$, then find an equation for the plane containing $P$, $Q$, and $R$.
(8) Suppose that a function $f$ is defined by $f_x = xf + y$, $f_y = f + 2$, $f(-1,2) = 3$. Use a tangent plane to approximate $f(-.5,1.6)$.
(9) Describe a real-world function $f(x, y)$, for which $f_x$ is negative, $f_y$ is positive, and the units on $f_x$ and $f_y$ are dollars per person and dollars per mile, respectively.
(10) If $f(x,y) = \frac{\sin xy + 2^x}{\ln y \cdot \arctan y}$, find $\nabla f$.
(11) Let $f(x,y) = x^2 y^2 + 3$. Use the limit definition of partial derivative to find $f_y$.
(12) Suppose that $\vec{u}$ and $\vec{v}$ have the same length, and that $\vec{u}$ points north and $\vec{v}$ points west. In which directions do the following vectors point? $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $\vec{v} - \vec{u}$, $\vec{v} + 1000\vec{u}$.
(13) A line contains the points (1,2,3) and (3,0,-3). Find a unit vector parallel to the line.
(14) Given $f(x,y) = x + y^2$. Find the equation of the tangent plane at the point (1,1,2).
(15) Find the equation of another plane which is orthogonal to the one you found in (14).

(Adapted from Exam 1 Study Guide, Math 224, Spring 2005, Prof. Don Lawrence)