Multivariable Calculus

Math 212 Spring 2006 — Fowler 112 MWF 8:30pm - 9:25am

 — http://faculty.oxy.edu/ron/math/212/06/

Class 21: **Friday March 24**

SUMMARY Extrema of Multivariable Functions, continued
CURRENT READING Williamson & Trotter, Section 6.4
HOMEWORK #20 Williamson & Trotter, page 298: **7, 8, 12, 13**. EXTRA CREDIT page 298: 15.

Algorithm for Multivariable Optimization

Given a function $f(\vec{x})$ defined on a region R in $\mathbb{R}^n$, compare the values of f at the following points:

(a) Critical points of f in the interior of R, where $\vec{\nabla} f = \vec{0}$

(b) Points on the Boundary of R

1. EITHER: Find a parametric representation $\vec{g}$ for the boundary of R, in which case we have a new optimization problem with the composite function $\vec{f}(\vec{g})$ defined on a set of one lower dimension,
2. OR: Use the Lagrange multiplier method by solving the system $\vec{\nabla} f = \sum_{k=1}^{n} \lambda_k \vec{\nabla} g_k$ where g_k are the functions representing the n constraints.

EXAMPLE 1

Suppose the function $f(x, y, z) = x + y + z$ is restricted to the intersection of the surfaces $x^2 + y^2 = 1$ and $z = 2$. Find the maximum and minimum value of the objective function f subject to the two given constraints.

$\boxed{\textbf{Exercise 1}}$

Williamson & Trotter, page 292, #25. A rectangular box with square base and no top is to contain volume V. Find the dimensions that yield the minimum cost if material for one side costs twice as much as for the other three sides and material for the base costs three time as much for the less expensive sides.

$\boxed{\text{GROUPWORK}}$

Williamson & Trotter, page 293, #37. (a) A rectangular shed with an open front and no flooring is to be built to shelter 108 cubic feet. If the roof material costs twice as much as the material for the three walls, what dimension will be the least expensive?

(b) How would the answer change if roofing costs the same as the walls?

$\boxed{\text{EXAMPLE 2}}$

What is the shortest distance from the surface $xy + 3x + z^2 = 9$ to the origin?