Math 212 Spring 2006

BONUS Quiz **11** **Multivariable Calculus**

Name: _____

Date: _____ **Monday April 24**
Time Begun: _____ Ron Buckmire
Time Ended: _____

Topic : Div, Grad and Curl

The idea behind this quiz is to provide you with another opportunity to illustrate your facility with the differential operators of vector fields.

Reality Check:

EXPECTED SCORE : _____**/10** ACTUAL SCORE : _____**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/212/06/`.
1. Once you open the quiz, you have as much time as you like to complete it, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. **Quizzes with loose sheets will not be graded.**
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Wednesday April 26**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 212 Spring 2006 **SHOW ALL YOUR WORK** **BONUS** Quiz Eleven

Consider the **Maxwell Field Equations** for a region in $\mathbb{R}^3$ containing no charge or current:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

where $\vec{B}(\vec{x}, t)$ is the magnetic field and $\vec{E}(\vec{x}, t)$ is the electric field and c is the speed of light. Given the vector identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$ manipulate the Maxwell Field Equations to obtain the following results:

a. (*5 points.*) Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$.

b. (*5 points.*) Show that $\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$ (This is a very famous equation known as **The Wave Equation**.)