Test 3: Multivariable Calculus

Math 212
Prof. Ron Buckmire

Friday April 28 2006
8:30pm-9:25am

Name: _____

Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. Notice which skills are designed to be tested on each question. This is a one hour, limited-notes (1 page), closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		30
2		40
3		30
BONUS		10
Total		**100**

1. Vector Fields, Double Integrals, Triple Integrals. *30 points.*
SKILLS: ANALYSIS/CRITICAL THINKING, VERBAL EXPRESSION.

TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once. .

(a) *10 points.* **TRUE or FALSE?** "There exists a non-zero vector field in $\mathbb{R}^3$ which has both zero curl and zero divergence."

(b) *10 points.* **TRUE or FALSE?** "Every triple integral can be written as a double integral."

(c) *10 points.* **TRUE or FALSE?** "Every double integral can be written as a triple integral."

2. Line Integration, Iterated Integration, Multiple Integration. (*40 points.*)
SKILLS: VISUALIZATION, COMPUTATION.

(a) (*10 points.*) Consider the path γ to be the straight line from the point (x_1, y_1) to the point (x_2, y_2). Write down a parametrization of this path and then show that the value of the line integral $\int_\gamma -\frac{y}{2}\, dx + \frac{x}{2}\, dy = \frac{1}{2}(x_1 y_2 - x_2 y_1)$. [**HINT: be very meticulous about maintaining all the subscripts during this calculation!**]

(b) (*10 points.*) Consider a triangular path Γ formed in the first quadrant by first moving from the origin horizontally a units to the right along the x-axis and then from this point moving upwards diagonally to the point b units on the y-axis above the origin and then down vertically back to the origin. Draw a picture of this path, labeling the coordinates of the vertices. Use your result from part(a) to help you in evaluating $\int_\Gamma -\frac{y}{2}\, dx + \frac{x}{2}\, dy$. [**HINT: you should not have to actually DO any integration in this problem!**]

(c) (*10 points.*) Considering Green's Theorem, write down a double integral which is equal in value to the line integral computed in part (b). Reverse the order of integration and evaluate this integral. [**HINT: you should already know what the answer to this problem is!**]

(d) (*10 points.*) The equation of the plane which intersects the x-axis at a, the y-axis at b and the z-axis at c is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. Show that the volume of the region bounded by this plane, the triangular region described in (b) and the $x = 0$ and $y = 0$ planes is equal to $\frac{1}{6}abc$.

3. Div, Grad, Curl, Green's Theorem, Fundamental Theorem of Line Integrals.
30 points.
SKILS: ANALYSIS/CRITICAL THINKING, COMPUTATION.]

(a) (*6 points.*)   By direct differentiation, compute **curl** $\vec{F} = \vec{\nabla} \times \vec{\nabla} f$ for $\vec{F}(x, y) = (F_1(x, y), F_2(x, y), 0)$.

(b) (*8 points.*) Consider a line integral of the gradient field $\vec{F} = \vec{\nabla} f$ on the boundary ∂R of an enclosed region R, i.e. $\oint_{\partial R} \vec{F} \cdot d\vec{x}$. What is the value of this line integral?

(c) (*8 points.*) Use Green's Theorem to convert your line integral in part (b) to a double integral (over a particular area). Write down this integral and evaluate it. What is the value of this double integral?
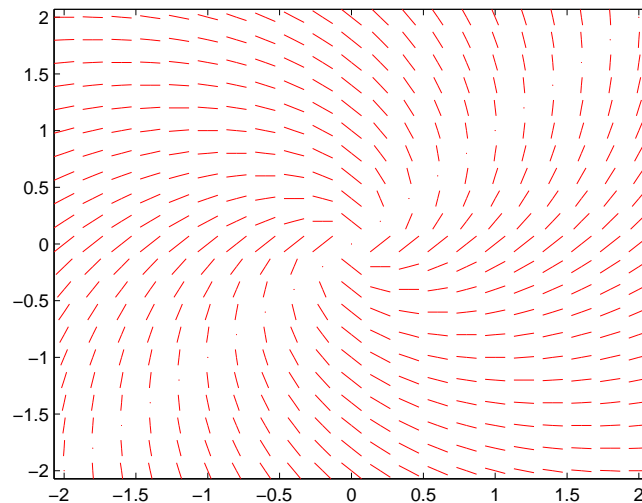
(d) (*8 points.*) What do your answers in (a), (b) and (c) tell you about the value of **curl grad** f?

BONUS QUESTION. Vector Fields and Vector Calculus. (*10 points.*)
SKILLS: VISUALIZATION, ANALYSIS, COMPUTATION.
Answer one of the questions below.

(I) Consider the vector field below. Discuss what properties (divergence, gradient, curl) of the field can be determined from the picture.

OR

(II) Show that **div curl** $\vec{F} = 0$ always. In other words, $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$ for every vector field $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$.