Test 1: Linear Systems

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Friday March 3 2006 2:30pm-3:25pm

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Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is a one hour, no-notes, closed book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."

No.	Score	Maximum
1		20
2		30
3		20
4		30
BONUS		10
Total	_	100

1. Span, Linear Independence, Rank. 20 points.

Consider the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

(a) (4 points.) Show that $rref(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) (4 points.) Given your knowledge of rref(A), what is the rank of the matrix A? **EXPLAIN YOUR ANSWER.**

(c) (4 points.) Given your knowledge of rref(A), what is the span of the columns of matrix A? **EXPLAIN YOUR ANSWER.**

(d) (4 points.) Given your knowledge of rref(A), discuss the linear independence of the columns of the matrix A. **EXPLAIN YOUR ANSWER.**

(e) (4 points.) Given your knowledge of rref(A), discuss whether the matrix A^{-1} exists. **EXPLAIN YOUR ANSWER.**

2. Dot product, magnitude, lengths. 30 points.

Suppose the dot product $\vec{u} \cdot \vec{v}$ is re-defined to be just the product of the lengths of the vectors \vec{u} and \vec{v} . Let's call this new dot product the **Buckmire product** and denote it

$$\vec{u} \circ \vec{v} = |\vec{u}||\vec{v}|$$

Discuss which of the following statements are true for all vectors \vec{u} , \vec{v} , $\vec{w} \in \mathbb{R}^n$ and all scalars $c \in \mathbb{R}$ under the **Buckmire product.**

(a) (6 points.) $\vec{u} \circ \vec{v} = \vec{v} \circ \vec{u}$

(b) (6 points.) $(c\vec{u}) \circ \vec{v} = c(\vec{u} \circ \vec{v})$

(c) (6 points.) $\vec{u} \circ (\vec{v} + \vec{w}) = \vec{u} \circ \vec{v} + \vec{u} \circ \vec{w}$

(d) (6 points.) $\vec{u} \circ \vec{u} \geq 0$

(e) (6 points.) $\vec{u} \circ \vec{u} = 0 \Leftrightarrow \vec{u} = \vec{0}$.

3. Matrix Operations. 20 points.

TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

Recall the zero matrix \mathcal{O} and identity matrix \mathcal{I} have particular properties in matrix arithmetic which often (but not always!) correspond to the properties the number zero and the number one that you know and love.

NOTE: A is assumed to be a generic (unknown) $m \times n$ matrix for every part below.

(a) 5 points. TRUE or FALSE? "If $A^2 = \mathcal{O}$ then $A = \mathcal{O}$."

(b) 5 points. TRUE or FALSE? "If $A = \mathcal{O}$ then $A^2 = \mathcal{O}$."

(c) 5 points. TRUE or FALSE? "If $A^2 = \mathcal{I}$ then $A = \mathcal{I}$."

(d) 5 points. TRUE or FALSE? "If $A = \mathcal{I}$ then $A^2 = \mathcal{I}$."

4.	Parametric	equations,	lines,	planes,	subspaces.	30 points.
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Consider the object described parametrically by x = t + 1, y = 2t - 3, z = 3t in \mathbb{R}^3 .

(a) (10 points.) Write down a system of three linear equations which has this object as its solution.

(b) (10 points.) What is the dimension of this object? What is the geometric interpretation of this solution to your linear system in **(a)**? Write down a vector equation describing this object.

(c) (10 points.) Is this object a subspace of \mathbb{R}^3 ? Prove your answer!

BONUS QUESTION. Linear Independence, Dependence, Inverse. (10 points.)

If possible, write down five different 3×3 matrices each one which has one of the following properties:

- (i) MATRIX A: The rows are linearly independent but the columns are linearly independent.
- (ii) MATRIX B: The rows are linearly dependent but the columns are linearly independent.
- (iii) MATRIX C: The rows are linearly independent but the columns are linearly dependent.
- (iv) MATRIX D: The rows are linearly dependent but the columns are linearly dependent.
- (v) MATRIX E: The transpose of the matrix equals the inverse of the matrix.

EXPLAIN YOUR ANSWER THOROUGHLY. EXTRA CREDIT POINTS ARE HARD TO GET.