Report on Test 1

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Point Distribution (N=17)

Range	100+	90+	85+	80+	75+	70+	65+	60+	55+	50+	45+	40+	40-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	0	2	2	2	2	2	1	3	1	1	0	1	0

Summary Overall class performance was surprising (to me); it was quite bimodal. The mean score was 72. There were 9 out of 17 above the average, and 8 of 17 below it.

#1 Multivariable Functions, Differentiation, Point Sets. This is a **TRUE or FALSE** question. Since all three questions start with the statement "There does not exist..." if there does exist such an object the statement will be False. **(a)** FALSE. A vector function of a scalar function has its derivative which is also a vector function of a scalar variable. So, then all one has to think of is what function has a derivative equal to itself? That would be $f(x) = 0$ or $f(x) = e^x$. Therefore, a vector function with these functions as component functions would have a derivative equal to itself, i.e. $\vec{f}(t) = (e^t, e^t, e^t)$ with $\dfrac{d\vec{f}}{dt} = (e^t, e^t, e^t)$. **(b)** TRUE. Since the derivative of a scalar function of a vector variable is a **gradient vector**, there is no way a vector can equal a scalar. **(c)** FALSE. Some students misread this question talking about a "point set" as referring to a set which only contains a single point. That was not my intention. All sets in $\mathbb{R}^n$ consist of points, and are known as point sets. Examples of sets which are both open and closed are the empty set and the set with all points in $\mathbb{R}^n$, i.e. $\mathbb{R}^n$ itself. A set consisting of a single point is NOT open but is closed.

#2 Analytic Geometry in $\mathbb{R}^n$, Lines, Planes. This question starts off familiar and then gets more theoretical. The idea was to think about the intersection of two hyperplanes in $\mathbb{R}^n$ completely generally. But you see the same question asked in four different spaces. **(a)** Whenever you have a problem involving $\mathbb{R}^2$ you gotta think "yay!" because you can draw a picture. So, clearly the two objects intersecting are 1-dimensional lines intersecting at a single point, a zero-dimensional object, at the point (1,0). **(b)** In $\mathbb{R}^3$ you have the problems of two planes intersecting each other. Clearly, if they are not parallel, these two 2-d objects will intersect in a line (1-d object). To determine the equation of the line one needs to use **both** of the given linear equations. **(c)** Thinking in $\mathbb{R}^4$ is a new experience for you, but it should follow from the previous two problems that one is having two three-dimensional objects in $\mathbb{R}^4$, known as hyperplanes intersect with each other to produce a 2-d object, which is often simply called a plane or a "2-plane." **(d)** The last question is the ultimate generalization of the problem; the intersection of two $(n-1)-$dimensional objects (in $\mathbb{R}^n$) called hyperplanes should produce an $(n-2)-$dimensional object. Thus it will require a linear combination of $n-2$ linearly independent vectors in $\mathbb{R}^n$. Again, by seeing how the two given linear combine to constrain the n variables allows one to obtain the vector equation.

#3 Partial Derivatives. Given the formula for $E = E(m, v)$ one can take partial derivatives of each expression using *all* the regular rules of differentiation. **(a)** Then by examining the form of the expressions one can see that $\dfrac{\partial E}{\partial m}$ will be positive. Similarly, by examining the expression $\dfrac{\partial E}{\partial v}$ one can see that it will also remain positive. Then you can discuss what the physical interpretation of these two quantities being positive implies about the relationship between rate of change of Energy with respect to mass (speed constant), and the rate of change of Energy with repect to speed (mass constant) for a moving body.