Multivariable Calculus

Math 212 Fall 2005 — Fowler 307 MWF 9:30pm - 10:25am

 — http://faculty.oxy.edu/ron/math/212/05/

Class 30: **Wednesday December 7**

SUMMARY Conservative Vector Fields
CURRENT READING Williamson & Trotter, §9.2

THEOREM

All gradient fields are conservative vector fields. All conservative vector fields have zero curl. All gradient fileds have zero curl.

THEOREM: properties of conservative vector fields

Let $\vec{F}$ be a continuous vector field defined in a polygonally connected open set D in $\mathbb{R}^n$. THEN *each* of the following three statements implies the other two.

(a) The integral $\vec{F}(\vec{x})$ over every piecewise smooth path from $\vec{x}_1$ to $\vec{x}_2$ in D has the same value, and we can write it as $\displaystyle\int_{\vec{x}_1}^{\vec{x}_2} \vec{F}(\vec{x}) \cdot d\vec{x} = \int_{\vec{x}_1}^{\vec{x}_2} \vec{\nabla} f(\vec{x}) \cdot d\vec{x} = f(\vec{x}_2) - f(\vec{x}_1)$.

(b) The integral over every piecewise smooth closed path γ in D is **zero**. In other words $\oint_\gamma \vec{F} \cdot d\vec{x} = \oint_\gamma \vec{\nabla} f \cdot d\vec{x} = 0$

(c) There is a continously differentiable function $f : D \to \mathbb{R}$ such that $\vec{F}$ is the gradient of f, i.e. $\vec{\nabla} f = \vec{F}$ for all $\vec{x}$ in D.

THEOREM

IF $\vec{F} : \mathbb{R}^n \to \mathbb{R}^n$ is a continuously differentiable gradient field, then $\vec{F}_{\vec{x}}$, the Jacobian matrix of $\vec{F}$ is **symmetric**. In other words $\dfrac{\partial F_i}{\partial x_j} = \dfrac{\partial F_j}{\partial x_i}$ for all $i, j = 1, 2, \ldots, n$.

EXAMPLE 1

Williamson & Trotter, page 418, #3. Is $\vec{F}(x, y) = (x - y, x + y)$ a gradient field?

Exercise 2

Williamson & Trotter, page 418, #4. Is $\vec{G}(x, y, z) = (y, z, x)$ a gradient field?

EXAMPLE 2

Williamson & Trotter, page 418, #10. Find a field potential for the given field.
$\vec{F}(x, y) = (2xy, x^2 + z^2, 2yz)$.

Exercise 2

Williamson & Trotter, page 418, #11. Find a field potential for the given field.
$\vec{G}(x, y) = (y\cos(xy), x\cos(xy))$.

GroupWork

Williamson & Trotter, page 418, #14. Consider the vector field $\vec{F}$ which is the gradient of the **Newtonian potential** $f(\vec{x}) = -|\vec{x}|^{-1}$ for nonzero $\vec{x}$ in $\mathbb{R}^3$. Find the work done in moving a particle from $(1, 1, 1)$ to $(-2, -2, -2)$ along a smooth curve lying in the domain of $\vec{F}$.