Multivariable Calculus

Math 212 Fall 2005 Fowler 307 MWF 9:30pm - 10:25am
 http://faculty.oxy.edu/ron/math/212/05/

Class 9: **Monday September 26**

SUMMARY Visualizing Multivariable Functions
CURRENT READING Williamson & Trotter, Section 4.2
HOMEWORK Williamson & Trotter, page 192: # 1, 4, 9, 10, 18

DEFINITION

The **graph** of a scalar function of a vector variable $f(\vec{x})$ is defined to be the set of ordered pairs $(\vec{x}, f(\vec{x}))$ where $\vec{x}$ is in the domain of f. In this case we say that the graph of f is **explicitly** represented by f.

In practice the only scalar functions of a vector function that we can really get a good handle on visually are either of the type $f : \mathbb{R} \to \mathbb{R}$ or $f : \mathbb{R}^2 \to \mathbb{R}$. These are $(x, f(x))$ and $(x, y, f(x, y))$ respectively. We know all about the first case so we will be concentrating on the second case, which are often called **surfaces** and denoted $z = f(z, y)$ so that the ordered pair looks like (x, y, z).

EXAMPLE 1

Graph the function $f(x, y) = 1 - x^2 - y^2$.

DEFINITION

The **level set** of a scalar function of a vector variable $f(\vec{x})$ is defined to be the set of values $\vec{x}$ in the domain of f such that $f(\vec{x}) = k$.

EXAMPLE 2

Describe the level sets of $f(x, y) = 1 - x^2 - y^2$.

Computer Generated Graphing

Go to the website `http://hypatia.math.uri.edu/ bkaskosz/flashmo/tools/`

GroupWork

Use the appropriate program to generate graphs of

(a) $f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ on $-\pi \le x \le \pi, \quad -\pi \le y \le \pi$

(b) $x = \cos(2t), y = 3\sin(2t), z = t/4$ for $0 \le t \le 20\pi$

(c) $x = t^5, y = t^2$ for $0 \le t \le 2$

Exercise

Williamson & Trotter, page 192, #2. Consider the function $g(x,) = \ln(x+y)$.

(a) Describe the domain of g, making it as large as possible.

(b) For what values of (x,y) does the graph of g lie above the xy-plane?

(c) Describe the image of g

Exercise

Williamson & Trotter, page 192, #13. Describe the $k = 0$ level set of $f(x,y,z) = xyz$.