Multivariable Calculus

Math 212 Fall 2005 Fowler 307 MWF 9:30pm - 10:25am

http://faculty.oxy.edu/ron/math/212/05/

Class 6: **Friday September 16**

SUMMARY Review of Linear Systems, Part 2: Matrix Operations
CURRENT READING Williamson & Trotter, Section 2.2, 2.3 and 2.4
HOMEWORK Williamson & Trotter, page 69: # 3, 11, 14; page 73: 4, 11 ; page 80: 1, 2, 3, 4 **Extra Credit page 81: 46, 50**

Matrix Multiplication

An $m \times n$ matrix multiplies an $n \times p$ matrix to produce an $m \times p$ product.

Properties of Matrix Multiplication

$(A+B)C = AC + BC$ (Right Distributive Law)
$C(A+B) = CA + CB$ (Left Distributive Law)
$(tA)(B) = t(AB) = A(tB)$ (Scalar Commutativity Law)
$A(BC) = (AB)C$ (Associative Law)
$AB \neq BA$ (Matrix Multiplication is not necessarily Commutative)

Linearity of Matrix Multiplication

$A(s\vec{u} + t\vec{v}) = sA\vec{u} + tA\vec{v}$

Theorem

Every solution of the linear system $A\vec{x} = \vec{b}$ has the form $\vec{x} = \vec{x}_h + \vec{x}_p$ where $\vec{x}_p$ is a particular solution of the system and $\vec{x}_h$ is a solution of the homogeneous equation $A\vec{x} = \vec{0}$

EXAMPLE 1

Give $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$. Find the solutions of $A\vec{x}_h = \vec{0}$ and $A\vec{x}_p = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$ and show that $\vec{x} = \vec{x}_p + \vec{x}_h$ is another solution of the non-homogeneous system.

Theorem

A homogeneous system $A\vec{x} = \vec{0}$ has infinitely many non-zero solutions if it has more variables than equations (i.e. $n > m$). It also has infinitely many non-zero solutions if the equivalent reduced system $R\vec{x} = \vec{0}$ has more variables than nontrivial equations.

DEFINITION

A **reduced matrix R** can be derived from the coefficient matrix A by applying elementary row operations. A reduced matrix R has the properties: **(i)** that every column containing a **leading entry** or pivot is zero except for that element and **(ii)** every leading entry (or pivot) equals 1.

EXAMPLE 2

Which of the following matrices are in reduced form?

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DEFINITION

A subset of $\mathbb{R}^n$ is called a k-**plane** if it consists of all points $\vec{x} = t_1\vec{u}_1 + t_2\vec{u}_2 + \ldots + t_k\vec{u}_k + \vec{v}$ where $\vec{v}$ is a fixed vector and the vectors $\vec{v}_i$ are linearly independent fixed vectors and t_i are real parametric variables. A **hyperplane** is another name for a $(n-1)$-plane, i.e. the solution set of a single linear equation in $\mathbb{R}^n$.

EXAMPLE 3

Williamson & Trotter, page 73, #1. Solve the equation $w+3x-2y+z = 3$ by expressing the solutions as a 3-plane in $\mathbb{R}^4$.

EXERCISE 1

Williamson & Trotter, page 73, #10. Show that the follwing system's solution form a line or 1-plane containing $\vec{0}$ in $\mathbb{R}^3$. $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$