Multivariable Calculus

Math 212 Fall 2005 Fowler 111 MWF 9:30pm - 10:25am

 http://faculty.oxy.edu/ron/math/212/05/

Class 2: **Wednesday September 7**

SUMMARY Parametric Equations of Lines and Planes
CURRENT READING Williamson & Trotter, Section 1.3
HOMEWORK Williamson & Trotter, §1.3 # 1, 2, 5, 8, 9, 11, 12, 15, 17, 19

Lines

Given two points in space (x_1, y_1) and (x_2, y_2) the general form of the equation of the line passing through them is given by:

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) + y_1$$

EXERCISE 1

Therefore, the equation of the line through the points $(1, 2)$ and $(-1, 1)$ is ____________________.

Using Vectors

What if we represent the points in space by position vectors $\vec{x}_1$ and $\vec{x}_2$ instead?
Then the line which goes through these two points can be represented by the vector equation

$$\vec{x} = \vec{x}_1 + t(\vec{x}_2 - \vec{x}_1)$$

where t is a parametric variable. NOTE: when $t = 0$, $\vec{x}$ equals the first point at $\vec{x}_1$ and when $t = 1$, $\vec{x}$ equals the second point $\vec{x}_2$. The left hand side $\vec{x}$ represents all the points (x, y) on the line connecting the points represented by $\vec{x}_1$ and $\vec{x}_2$.

EXAMPLE 1

Let's use our new formula for the equation of a line given $\vec{x}_1 = (1, 2)$ and $\vec{x}_2 = (-1, 1)$.

Are these points in space making up the line in this example the same set of points obtained in Exercise 1?
In other words is the parametric equation equivalent to the Cartesian equation we derived above and know and love?
Let's check. The vector equation from our example ends up producing these two parametric equations:

$$\begin{aligned} x(t) &= 2t + 1 \\ y(t) &= t + 2 \end{aligned}$$

To convert our parametric equations into the one cartesian equation, solve each equation for t and equate them to each other. What relationship do you obtain between x and y?

EXERCISE 2

Find a parametric representation for the line in $\mathbb{R}^3$ through $(-1,1,0)$ and $(2,2,1)$. [HINT: Use the vector formula!]

DEFINITION

Two lines are said to be **parallel** if they have representations $t\vec{a}_1 + \vec{b}_1$ and $t\vec{a}_2 + \vec{b}_2$ and the vectors $\vec{a}_1$ and $\vec{a}_2$ are parallel.

EXERCISE 3

Williamson & Trotter, p. 23, #10. Find out whether the two lines $t(4,2,2) + (2,0,1)$ and $t(2,1,1) + (2,2,1)$ are the same, and if they aren't whether they are parallel.

Planes

The equation of a plane through the origin is given by

$$\vec{x} = t_1\vec{x}_1 + t_2\vec{x}_2$$

where t_1 and t_2 are parametric variables and $\vec{x}_1$ and $\vec{x}_2$ are linearly independent.
In other words, a plane through the origin is defined as the set of all possible **linear combinations** of two vectors.
If the plane is not through the origin, it can be expressed as

$$\vec{x} = t_1\vec{x}_1 + t_2\vec{x}_2 + \vec{x}_3$$

where $\vec{x}_1$ and $\vec{x}_2$ are not multiples of each other (i.e. linearly independent) and all three vectors $\vec{x}_1, \vec{x}_2, \vec{x}_3$ are fixed.

DEFINITION

A **linear combination** of vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots \vec{v}_n$ is the set of all vectors produced by $\displaystyle\sum_{i=1}^{n} c_i\vec{v}_i = c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + \vec{v}_n$ where $c_i \in \mathbb{R}$.

DEFINITION

A set of vectors is said to be **linearly independent** if none of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots \vec{v}_n$ can be written as a linear combination of the others. In other words, the only solution to the linear equation $\sum_{i=1}^{n} c_i\vec{v}_i = c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + \vec{v}_n = 0$ is $c_i = 0$

EXAMPLE 2

Williamson & Trotter, page 23, #18. Find the equation of the plane passing through the points $(1,1,0)$, $(-3,0,2)$ and $(2,4,7)$.