Math 212 Fall 2005

BONUS Quiz **8** **Multivariable Calculus**

Name: _____

Date: _____ **Wednesday November 30**
Time Begun: _____ Ron Buckmire
Time Ended: _____

Topic : Div, Grad and Curl

The idea behind this quiz is to provide you with another opportunity to illustrate your facility with the differential operators of vector fields.

Reality Check:

EXPECTED SCORE : _____**/10** ACTUAL SCORE : _____**/10**

Instructions:

0. Please look for a hint on this quiz posted to `http://faculty.oxy.edu/ron/math/212/05/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday December 5, in class**. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Consider the **Maxwell Field Equations** for a region in $\mathbb{R}^3$ containing no charge or current:

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

where $\vec{B}(\vec{x}, t)$ is the magnetic field and $\vec{E}(\vec{x}, t)$ is the electric field and c is the speed of light. Given the vector identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$ manipulate the Maxwell Field Equations to obtain the following results:

a. (*5 points.*) Show that $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$.

b. (*5 points.*) Show that $\nabla^2 \vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$ (This is a very famous equation known as **The Wave Equation**.)