Math 212 Fall 2005

BONUS Quiz **3** — Multivariable Calculus

Name: _____

Date: _____

Time Begun: _____

Time Ended: _____

Friday September 30

Ron Buckmire

Topic : Partial Derivatives

The idea behind this quiz is to provide you with an opportunity to illustrate your facility with partial differentiation.

Reality Check:

EXPECTED SCORE : _____/**10**   ACTUAL SCORE : _____/**10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/212/05/`.

1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.

2. You may use the book or any of your class notes. You must work alone.

3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.

5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.

6. Relax and enjoy...

7. **This quiz is due on Monday October 3**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Consider the Kortweg-de Vries equation (usually called the KdV equation) $u_t + uu_x + u_{xxx} = 0$ where the solution $u(x,t)$ represents the shape of a travelling water wave known as a **soliton.**

The general form of the soliton is $u(x,t) = 3c\,\text{sech}^2(\frac{\sqrt{c}}{2}(x-ct))$ where $\text{sech}(x)$ is the hyperbolic secant function and c is a parameter representing the wave speed. You may recall that hyperbolic trigonometric functions are defined as follows: $\cosh(x) = \dfrac{e^x + e^{-x}}{2}, \sinh(x) = \dfrac{e^x - e^{-x}}{2}$ and $\text{sech}(x) = \dfrac{1}{\cosh(x)}$.

1. (*2 points*) Show that $u_t + cu_x = 0$

2. (a) (*4 points*) Think of the soliton as a surface in x and t variables. Find the equation of the tangent plane to this surface at $(0,0)$.

2. (b) (*4 points*) Show that the given form of the soliton obeys Clairault's Theorem.

(**NOTE:** $\frac{d}{dx}\textbf{sech}(x) = -\textbf{sech}(x)\textbf{tanh}(x)$ **and** $\frac{d}{dx}\textbf{tanh}(x) = \textbf{sech}^2(x)$)