MATH 212 STUDY GUIDE SOLUTIONS (BUCKMIRE)

5. $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

$$\vec{\nabla} f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} 6x^2 + y^2 + 10x \\ 2xy + 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2y(x+1) = 0 \Rightarrow y = 0$ or $x = -1$

when $y = 0$, $6x^2 + 10x = 0$ when $x = 0$ or $3x + 5 = 0$, $x = -5/3$

when $x = -1$, $6 - 10 + y^2 = 0 \Rightarrow y = \pm 2$

Critical Points are

$(0,0)$ $(-5/3, 0)$ $(-1, 2)$ $(-1,-2)$

$f_{xx} = 12x + 10$ $f_{xy} = 2y$ Recall

$f_{yy} = 2x + 2$ $f_{yx} = 2y$ $D = f_{xx} f_{yy} - f_{xy} f_{yx}$

At $(0,0)$ $D = 10 \cdot 2 - 0^2 = 20 > 0$ with $f_{xx} > 0$, $f_{yy} > 0$

LOCAL MINIMUM

At $(-5/3, 0)$ $D = (-10)(-\frac{4}{3}) - 0^2 > 0$ with $f_{xx} < 0$, $f_{yy} < 0$

LOCAL MAXIMUM

At $(-1, 2)$ $D = (-2) \cdot 0 - (-4)^2 = -16 < 0$

SADDLE POINT

At $(-1,-2)$ $D = (-2) \cdot 0 - (4)^2 = -16 < 0$

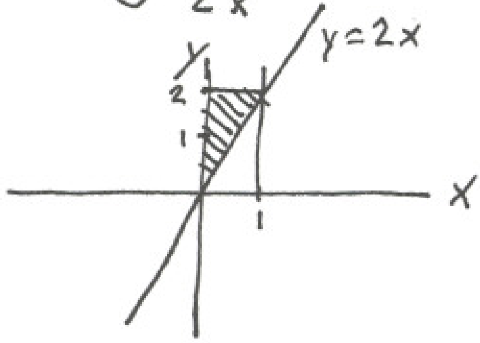
SADDLE POINT

Clearly $(0,0,0)$ is not a GLOBAL MIN since $(-2,0)$
The function will be negative

$(-5/3, 0)$ is not location of GLOBAL MAX since it's easy for the function to become very large.

MATH 212 EXAM 2 STUDY GUIDE

6. $\displaystyle\int_0^1\int_{2x}^2 \cos(1+y^2)\,dy\,dx = \int_0^2\int_0^{y/2} \cos(1+y^2)\,dx\,dy$



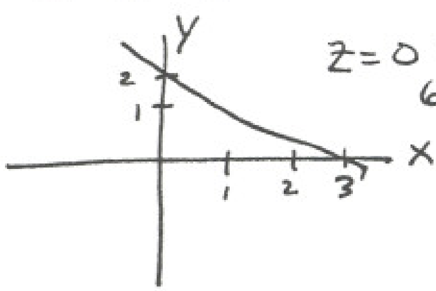
$\displaystyle = \int_0^2 \cos(1+y^2)\,\frac{y}{2}\,dy$

$\displaystyle = \frac{1}{4}\sin(1+y^2)\Big|_0^2$

$\displaystyle = \frac{1}{4}\sin(5) - \frac{1}{4}\sin(1)$

7. $z = 6-2x-3y$

$\displaystyle\int_0^2\int_0^{\frac{1}{2}(6-3y)}\int_0^{6-2x-3y} dz\,dx\,dy = \int_0^3\int_0^{2-\frac{2}{3}x} 6-2x-3y\,dy\,dx = V$



$z = 0 = 6-2x-3y$

$6 = 2x+3y$

$2-\frac{2x}{3} = y$

$3-\frac{3}{2}y = x$

$\displaystyle V = \int_0^2\int_0^{3-\frac{3}{2}y} 6-2x-3y\,dx\,dy$

$=$

8. $f(x,y) = 3x-2y$

$g(x,y) = x^2+2y^2-44 = 0$

$\vec{\nabla} f = (3,-2)$

$\vec{\nabla} g = (2x,4y)$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$g = 0$

$f(2,-6) = 3\cdot 2-2(-6) = 6+12$
$= 18$

$f(-2,6) = 3\cdot(-2)-2(6) = -6$
-12
$= -18$

$3 = \lambda 2x$
$-2 = \lambda 4y$
$x^2+2y^2 = 44$

$\displaystyle\frac{3}{2x} = \frac{-2}{4y} \Rightarrow 12y = -4x$
$3y = -x$

$9y^2+2y^2 = 44$
$11y^2 = 44$
$y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow x = \mp 6$

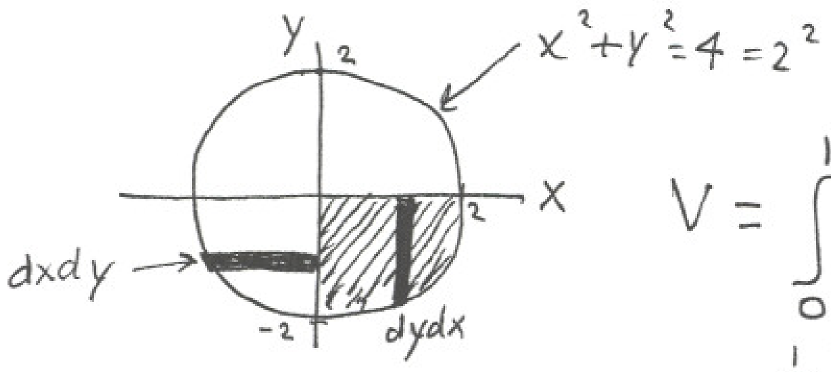
$(2,-6,18)$ is a MAX

$(-2,6,-18)$ is a MIN

MATH 212 EXAM 2 STUDY GUIDE SOLUTIONS

9. $$\int_0^1 \int_{-2}^0 \int_{-\sqrt{4-y^2}}^0 \cos(x^2+y^2)\,dx\,dy\,dz = \int_0^1 \int_{-\pi}^{-\pi/2} \int_0^2 \cos(r^2)\,r\,dr\,d\theta\,d$$

cylindrical coordinate



$$V = \int_0^1 \int_{\pi}^{3\pi/2} \frac{\sin(r^2)}{2}\Big|_0^2 \,d\theta\,dz$$

$$= \int_0^1 \int_{\pi}^{3\pi/2} \frac{\sin(4)}{2}\,d\theta\,dz = 1 \cdot \frac{\pi}{2} \cdot \frac{\sin(4)}{2}$$

$$= \frac{\pi \sin(4)}{4}$$

10. $f(x,y) = x^2 + xy^4$

Clearly there is no GLOBAL MAX since

$$\lim_{x\to\infty} \lim_{y\to\infty} f(x,y) = \infty$$

There is no GLOBAL MIN since if $x<0$ and $y\to\infty$ then $f\to-\infty$.

11. (b) $xy - z^2 = xe^z - 1$ at $x=0, y=3, z=1$

$f(x,y,z) = xy - z^2 - xe^z + 1 = 0$

Using implicit differentiation

$\vec{x} = (x,y)$ $\quad \vec{y} = z$ $\quad \vec{y} = \vec{G}(\vec{x})$

$z = f(x,y)$

$G'(\vec{x}) = -[F_{\vec{y}}]^{-1} \vec{F}_{\vec{x}}$

$$G'(\vec{x}) = \begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix}$$

$F_y = -2z - xe^z$

At $(0,3,1)$

$$\vec{F}_{\vec{x}} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} y - e^z \\ x \end{pmatrix}$$

$F_y = -2$

$$F_{\vec{x}} = \begin{pmatrix} 3-e \\ 0 \end{pmatrix}$$

$$G'(\vec{x}) = -\frac{1}{-2}\begin{pmatrix} 3-e \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3-e}{2} \\ 0 \end{pmatrix}$$

10(a) Without using implicit differentiation

$$f(x,y,z) = c$$
$$f(\vec{x}) = c \qquad \vec{\nabla} f = \langle y - e^z, x, -2z - xe^z \rangle$$
$$\nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0) = 0 \qquad \vec{x}_0 = (0,3,1) \quad \vec{x} = (x,y,z)$$

$$(3-e, 0, -2) \cdot (x-0, y-3, z-1) = 0$$
$$(3-e)x + 0 \cdot (y-3) - 2(z-1) = 0$$
$$(3-e)x + 0 \qquad = +2z + 2$$
$$(3-e)x + 2 \qquad = +2z$$
$$\left(\frac{3-e}{+2}\right)x + 1 \qquad = z$$
$$\frac{\partial z}{\partial y} = 0 \qquad \frac{\partial z}{\partial x} = \frac{3-e}{2}$$

(Same answer as in (b)) Neat, huh?

11. $f(x,y,z) = x^2 + y^3 + 5yz \qquad \vec{\nabla} f = (2x, 3y^2 + 5z, 5y) = f'$

$$g(t) = (a(t), b(t), c(t)) \qquad a(0) = 4 \quad b(0) = 5 \quad c(0) = 6$$
$$a'(0) = 3 \quad b'(0) = 2 \quad c'(0) = 1$$
$$h(t) = f(g(t))$$

$$h'(0) = f'(g(0))\, g'(0) = \left. \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{dz}{dt} \right|_{t=0}$$
$$= f'(4,5,6) \cdot (3,2,1) = (2 \cdot 4) \cdot 3 + (3 \cdot 5^2 + 5 \cdot 6) \cdot 2 + (5 \cdot 5) \cdot 1$$
$$= 24 + 210 + 25$$
$$= 259$$