

How Students Learn

MATHEMATICS IN THE CLASSROOM

Committee on *How People Learn*, A Targeted Report for Teachers

M. Suzanne Donovan and John D. Bransford, *Editors*

Division of Behavioral and Social Sciences and Education

NATIONAL RESEARCH COUNCIL
OF THE NATIONAL ACADEMIES

THE NATIONAL ACADEMIES PRESS
Washington, D.C.
www.nap.edu

THE NATIONAL ACADEMIES PRESS • 500 Fifth Street, N.W. • Washington, D.C. 20001

NOTICE: The project that is the subject of this report was approved by the Governing Board of the National Research Council, whose members are drawn from the councils of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine. The members of the committee responsible for the report were chosen for their special competences and with regard for appropriate balance.

This study was supported by Award No. R215U990024 between the National Academy of Sciences and the U.S. Department of Education. Any opinions, findings, conclusions, or recommendations expressed in this publication are those of the author(s) and do not necessarily reflect the views of the organizations or agencies that provided support for the project.

Library of Congress Cataloging-in-Publication Data

National Research Council (U.S.). Committee on How People Learn, A Targeted Report for Teachers.

How students learn : history, mathematics, and science in the classroom / Committee on How People Learn, A Targeted Report for Teachers ; M. Suzanne Donovan and John D. Bransford, editors.

p. cm.

"Division of Behavioral and Social Sciences and Education."

Includes bibliographical references and index.

ISBN 0-309-07433-9 (hardcover) — ISBN 0-309-08948-4 (pbk.) —

ISBN 0-309-08949-2 (pbk.) — ISBN 0-309-08950-6 (pbk.) 1. Learning. 2.

Classroom management. 3. Curriculum planning. I. Donovan, Suzanne.

II. Bransford, John. III. Title.

LB1060.N38 2005

370.15'23—dc22

2004026246

Additional copies of this report are available from the National Academies Press, 500 Fifth Street, N.W., Lockbox 285, Washington, DC 20055; (800) 624-6242 or (202) 334-3313 (in the Washington metropolitan area); Internet, <http://www.nap.edu>

Printed in the United States of America.

First Printing, December 2004

Second Printing, October 2005

Copyright 2005 by the National Academy of Sciences. All rights reserved.

Suggested citation: National Research Council. (2005). *How Students Learn: Mathematics in the Classroom*. Committee on *How People Learn*, A Targeted Report for Teachers, M.S. Donovan and J.D. Bransford, Editors. Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.



Introduction

M. Suzanne Donovan and John D. Bransford

More than any other species, people are designed to be flexible learners and, from infancy, are active agents in acquiring knowledge and skills. People can invent, record, accumulate, and pass on organized bodies of knowledge that help them understand, shape, exploit, and ornament their environment. Much that each human being knows about the world is acquired informally, but mastery of the accumulated knowledge of generations requires intentional learning, often accomplished in a formal educational setting.

Decades of work in the cognitive and developmental sciences has provided the foundation for an emerging science of learning. This foundation offers conceptions of learning processes and the development of competent performance that can help teachers support their students in the acquisition of knowledge that is the province of formal education. The research literature was synthesized in the National Research Council report *How People Learn: Brain, Mind, Experience, and School*.¹ In this volume, we focus on three fundamental and well-established principles of learning that are highlighted in *How People Learn* and are particularly important for teachers to understand and be able to incorporate in their teaching:

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.
2. To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

3. A "metacognitive" approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

A FISH STORY

The images from a children's story, *Fish Is Fish*,² help convey the essence of the above principles. In the story, a young fish is very curious about the world outside the water. His good friend the frog, on returning from the land, tells the fish about it excitedly:

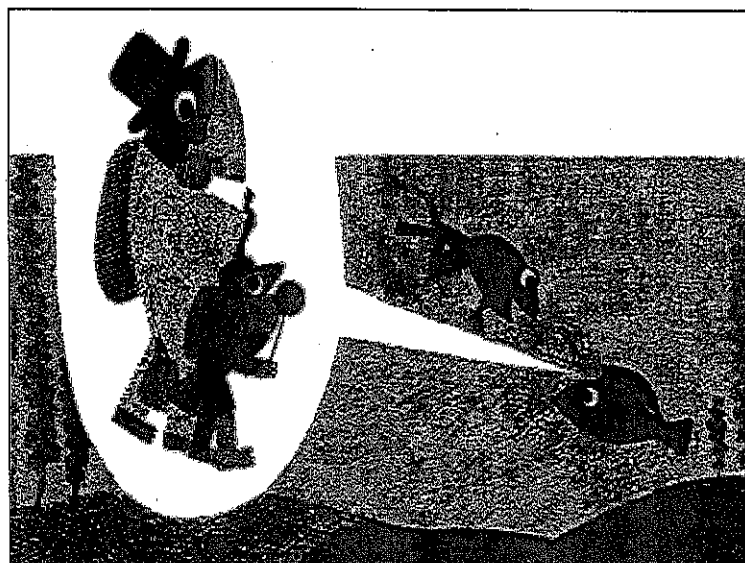
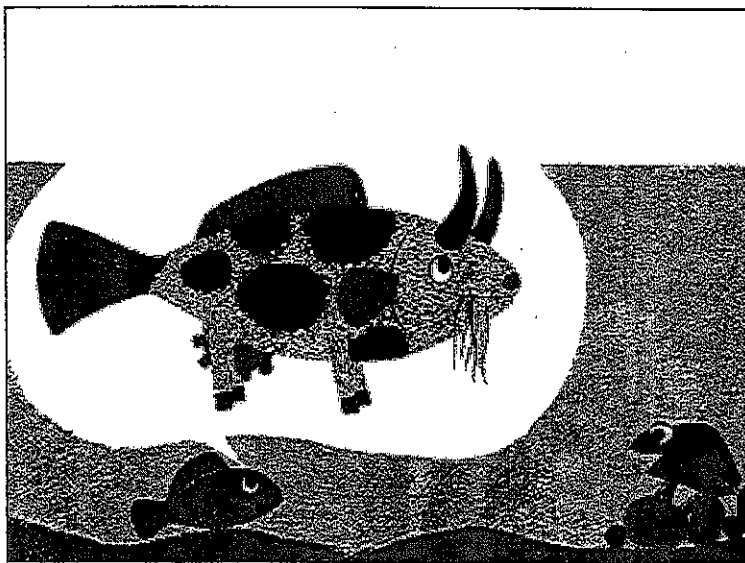
"I have been about the world—hopping here and there,"
said the frog, "and I have seen extraordinary things."

"Like what?" asked the fish.

"Birds," said the frog mysteriously. "Birds!" And he told the
fish about the birds, who had wings, and two legs, and
many, many colors. As the frog talked, his friend saw the
birds fly through his mind like large feathered fish.

The frog continues with descriptions of cows, which the fish imagines as black-and-white spotted fish with horns and udders, and humans, which the fish imagines as fish walking upright and dressed in clothing. Illustrations below from Leo Lionni's *Fish Is Fish* © 1970. Copyright renewed 1998 by Leo Lionni. Used by permission of Random House Children's Books, a division of Random House, Inc.





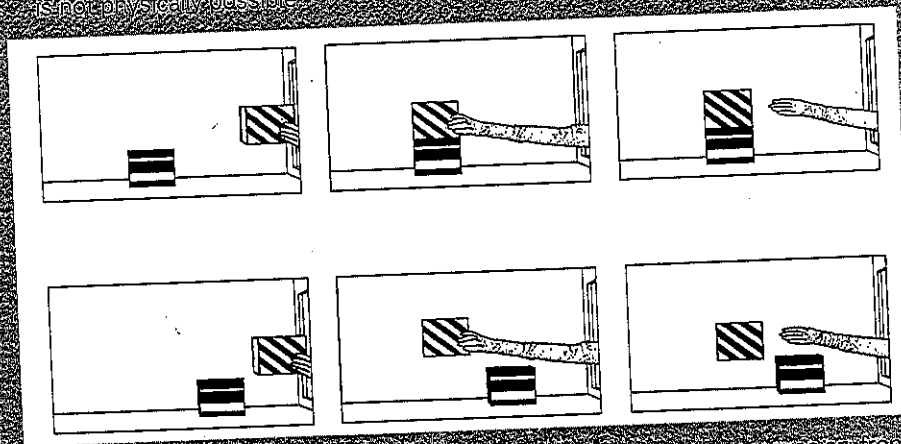
Principle #1: Engaging Prior Understandings

What Lionni's story captures so effectively is a fundamental insight about learning: *new understandings are constructed on a foundation of existing understandings and experiences.* With research techniques that permit the study of learning in infancy and tools that allow for observation of activity in the brain, we understand as never before how actively humans engage in learning from the earliest days of life (see Box 1-1). The understandings children carry with them into the classroom, even before the start of formal schooling, will shape significantly how they make sense of what they are

BOX 1-1 The Development of Physical Concepts in Infancy

Research studies have demonstrated that infants as young as 3 to 4 months of age develop understandings and expectations about their physical world. For example, they understand that objects need support to prevent them from falling to the ground; that stationary objects may be displaced when they come into contact with moving objects; and that objects at rest must be propelled into motion.

In research by Neelham and Ballageon, infants were shown a table on which a box rested. A gloved hand reached out from a window beside the table and placed another box in one of two locations: on top of the first box (the possible event) and beyond the box—directly in the path on which the box was suspended in midair. In this and similar studies, infants look reliably longer at the impossible events, suggesting an awareness and a set of expectations regarding what is and is not physically possible.



SOURCE: Neelham and Ballageon (1993). Reprinted with permission from Elsevier.

BOX 1-2 Misconceptions About Momentum

Andrea D. Sessa⁵ conducted a study in which he compared the performance of college physics students at a top technological university with that of elementary school children on a task involving momentum. He instructed both sets of students to play a computerized game that required them to direct a simulated object (a dvirturtle) so that it would hit a target and to do so with minimum speed at impact. Participants were introduced to the game and given a hands-on material that allowed them to apply a few taps with a wooden mallet to a ball on a table before they began.

D. Sessa found that both groups of students failed miserably at the task. Despite their training, college physics majors—just like the elementary school children—applied the force when the object was just below the target, failing to take momentum into account. Further investigation with one college student revealed that she knew the relevant physical properties and formulas and would have performed well on a written exam. Yet in the context of the game, she fell back on her untrained conceptions of how the physical world works.

taught. Just as the fish constructed an image of a human as a modified fish, children use what they know to shape their new understandings.

While prior learning is a powerful support for further learning, it can also lead to the development of conceptions that can act as barriers to learning. For example, when told that the earth is round, children may look to reconcile this information with their experience with balls. It seems obvious that one would fall off a round object. Researchers have found that some children solve the paradox by envisioning the earth as a pancake, a “round” shape with a surface on which people could walk without falling off.⁶

How People Learn summarizes a number of studies demonstrating the active, preconception-driven learning that is evident in humans from infancy through adulthood.⁷ Preconceptions developed from everyday experiences are often difficult for teachers to change because they generally work well enough in day-to-day contexts. But they can impose serious constraints on understanding formal disciplines. College physics students who do well on classroom exams on the laws of motion, for example, often revert to their untrained, erroneous models outside the classroom. When they are confronted with tasks that require putting their knowledge to use, they fail to take momentum into account, just as do elementary students who have had no physics training (see Box 1-2). If students’ preconceptions are not addressed directly, they often memorize content (e.g., formulas in physics), yet still use their experience-based preconceptions to act in the world.

Principle #2: The Essential Role of Factual Knowledge and Conceptual Frameworks in Understanding

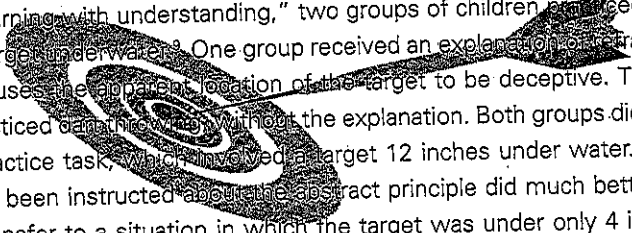
The *Fish Is Fish* story also draws attention to the kinds of knowledge, factual and conceptual, needed to support learning with understanding. The frog in the story provides information to the fish about humans, birds, and cows that is accurate and relevant, yet clearly insufficient. Feathers, legs, udders, and sport coats are surface features that distinguish each species. But if the fish (endowed now with human thinking capacity) is to understand how the land species are different from fish and different from each other, these surface features will not be of much help. Some additional, critical concepts are needed—for example, the concept of adaptation. Species that move through the medium of air rather than water have a different mobility challenge. And species that are warm-blooded, unlike those that are cold-blooded, must maintain their body temperature. It will take more explaining of course, but if the fish is to see a bird as something other than a fish with feathers and wings and a human as something other than an upright fish with clothing, then feathers and clothing must be seen as adaptations that help solve the problem of maintaining body temperature, and upright posture and wings must be seen as different solutions to the problem of mobility outside water.

Conceptual information such as a theory of adaptation represents a kind of knowledge that is unlikely to be induced from everyday experiences. It typically takes generations of inquiry to develop this sort of knowledge, and people usually need some help (e.g., interactions with “knowledgeable others”) to grasp such organizing concepts.⁸

Lionni's fish, not understanding the described features of the land animals as adaptations to a terrestrial environment, leaps from the water to experience life on land for himself. Since he can neither breathe nor maneuver on land, the fish must be saved by the amphibious frog. The point is well illustrated: learning with understanding affects our ability to apply what is learned (see Box 1-3).

This concept of learning with understanding has two parts: (1) factual knowledge (e.g., about characteristics of different species) must be placed in a conceptual framework (about adaptation) to be well understood; and (2) concepts are given meaning by multiple representations that are rich in factual detail. Competent performance is built on neither factual nor conceptual understanding alone; the concepts take on meaning in the knowledge-rich contexts in which they are applied. In the context of Lionni's story, the general concept of adaptation can be clarified when placed in the context of the specific features of humans, cows, and birds that make the abstract concept of adaptation meaningful.

BOX 1-3 Learning with Understanding Supports Knowledge Use in New Situations



In one of the most famous early studies comparing the effects of "learning a procedure" with "learning with understanding," two groups of children practiced throwing darts at a target underwater.⁸ One group received an explanation of refraction of light, which causes the apparent location of the target to be deceptive. The other group only practiced darts throwing without the explanation. Both groups did equally well on the practice task, which involved a target 12 inches under water. But the group that had been instructed about the abstract principle did much better when they had to transfer to a situation in which the target was under only 4 inches of water. Because they understood what they were doing, the group that had received instruction about the refraction of light could adjust their behavior to the new task.

This essential link between the factual knowledge base and a conceptual framework can help illuminate a persistent debate in education: whether we need to emphasize "big ideas" more and facts less, or are producing graduates with a factual knowledge base that is unacceptably thin. While these concerns appear to be at odds, knowledge of facts and knowledge of important organizing ideas are mutually supportive. Studies of experts and novices—in chess, engineering, and many other domains—demonstrate that experts know considerably more relevant detail than novices in tasks within their domain and have better memory for these details (see Box 1-4). But the reason they remember more is that what novices see as separate pieces of information, experts see as organized sets of ideas.

Engineering experts, for example, can look briefly at a complex mass of circuitry and recognize it as an amplifier, and so can reproduce many of its circuits from memory using that one idea. Novices see each circuit separately, and thus remember far fewer in total. Important concepts, such as that of an amplifier, structure both what experts notice and what they are able to store in memory. Using concepts to organize information stored in memory allows for much more effective retrieval and application. Thus, the issue is not whether to emphasize facts or "big ideas" (conceptual knowledge); both are needed. Memory of factual knowledge is enhanced by conceptual knowledge, and conceptual knowledge is clarified as it is used to help organize constellations of important details. Teaching for understanding, then, requires that the core concepts such as adaptation that organize the knowledge of experts also organize instruction. This does not mean that factual knowledge now typically taught, such as the characteristics of fish, birds, and mammals, must be replaced. Rather, that factual information is given new meaning and a new organization in memory because those features are seen as adaptive characteristics.

Principle #3: The Importance of Self-Monitoring

Hero though he is for saving the fish's life, the frog in Lionni's story gets poor marks as a teacher. But the burden of learning does not fall on the teacher alone. Even the best instructional efforts can be successful only if the student can make use of the opportunity to learn. Helping students become effective learners is at the heart of the third key principle: a "metacognitive" or self-monitoring approach can help students develop the ability to take control of their own learning, consciously define learning goals, and monitor their progress in achieving them. Some teachers introduce the idea of metacognition to their students by saying, "You are the owners and operators of your own brain, but it came without an instruction book. We need to learn how we learn."

"Meta" is a prefix that can mean after, along with, or beyond. In the psychological literature, "metacognition" is used to refer to people's knowledge about themselves as information processors. This includes knowledge about what we need to do in order to learn and remember information (e.g., most adults know that they need to rehearse an unfamiliar phone number to keep it active in short-term memory while they walk across the room to dial the phone). And it includes the ability to monitor our current understanding to make sure we understand (see Box 1-5). Other examples include monitoring the degree to which we have been helpful to a group working on a project.¹⁰

BOX 1-5 Metacognitive Monitoring: An Example

Read the following passage from a literary critic and pay attention to the strategies you use to comprehend it.

Transcendentalists were so wise and noble, all right, but how could they all Mother-shit themselves? My Boyfriends Away, Grace Humpal, review, Starbuck Press, will assume you're not going mad, but that's what I must be, was the editor's wife.

Most good readers have to read it several times in order to grasp the meaning of this passage. In contrast, poor readers tend to simply read it all the way through without pausing and asking if the passage makes sense. Needless to say, when asked to paraphrase the passage they fall short.

SOURCE: Whimbey and Whimbey (1975, p. 42)

In Lionni's story, the fish accepted the information about life on land rather passively. Had he been monitoring his understanding and actively comparing it with what he already knew, he might have noted that putting on a hat and jacket would be rather uncomfortable for a fish and would slow his swimming in the worst way. Had he been more engaged in figuring out what the frog meant, he might have asked why humans would make themselves uncomfortable and compromise their mobility. A good answer to his questions might have set the stage for learning about differences between humans and fish, and ultimately about the notion of adaptation. The concept of metacognition includes an awareness of the need to ask how new knowledge relates to or challenges what one already knows—questions that stimulate additional inquiry that helps guide further learning.¹¹

The early work on metacognition was conducted with young children in laboratory contexts.¹² In studies of "metamemory," for example, young children might be shown a series of pictures (e.g., drum, tree, cup) and asked to remember them after 15 seconds of delay (with the pictures no longer visible). Adults who receive this task spontaneously rehearse during the 15-second interval. Many of the children did not. When they were explicitly told to rehearse, they would do so, and their memory was very good. But when the children took part in subsequent trials and were not reminded to rehearse, many failed to rehearse even though they were highly motivated to perform well in the memory test. These findings suggest that the children had not made the "metamemory" connection between their rehearsal strategies and their short-term memory abilities.¹³

Over time, research on metacognition (of which metamemory is considered a subset) moved from laboratory settings to the classroom. One of the most striking applications of a metacognitive approach to instruction was pioneered by Palincsar and Brown in the context of "reciprocal teaching."¹⁴ Middle school students worked in groups (guided by a teacher) to help one another learn to read with understanding. A key to achieving this goal involves the ability to monitor one's ongoing comprehension and to initiate strategies such as rereading or asking questions when one's comprehension falters. (Box 1-5 illustrates this point.) When implemented appropriately, reciprocal teaching has been shown to have strong effects on improving students' abilities to read with understanding in order to learn.

Appropriate kinds of self-monitoring and reflection have been demonstrated to support learning with understanding in a variety of areas. In one study,¹⁵ for example, students who were directed to engage in self-explanation as they solved mathematics problems developed deeper conceptual understanding than did students who solved those same problems but did not engage in self-explanation. This was true even though the common time limitation on both groups meant that the self-explaining students solved fewer problems in total.

Helping students become more metacognitive about their own thinking and learning is closely tied to teaching practices that emphasize self-assessment. The early work of Thorndike¹⁶ demonstrated that feedback is important for learning. However, there is a difference between responding to feedback that someone else provides and actively seeking feedback in order to assess one's current levels of thinking and understanding. Providing support for self-assessment is an important component of effective teaching. This can include giving students opportunities to test their ideas by building things and seeing whether they work, performing experiments that seek to falsify hypotheses, and so forth. Support for self-assessment is also provided by opportunities for discussion where teachers and students can express different views and explore which ones appear to make the most sense. Such questioning models the kind of dialogue that effective learners internalize. Helping students explicitly understand that a major purpose of these activities is to support metacognitive learning is an important component of successful teaching strategies.¹⁷

Supporting students to become aware of and engaged in their own learning will serve them well in all learning endeavors. To be optimally effective, however, some metacognitive strategies need to be taught in the context of individual subject areas. For example, guiding one's learning in a particular subject area requires awareness of the disciplinary standards for knowing. To illustrate, asking the question "What is the evidence for this claim?" is relevant whether one is studying history, science, or mathematics. However, what counts as evidence often differs. In mathematics, for example, formal proof is very important. In science, formal proofs are used when possible, but empirical observations and experimental data also play a major role. In history, multiple sources of evidence are sought and attention to the perspective from which an author writes and to the purpose of the writing is particularly important. Overall, knowledge of the discipline one is studying affects people's abilities to monitor their own understanding and evaluate others' claims effectively.