Warm-Up

Question: How are the following integrals related? \[ \int e^{\tan(x)} \sec^2(x) \, dx \text{ and } \int 2xe^{x^2} \, dx \]

Answer: We can think of two different \( u \)-substitutions which will make these integrals identical.

Integral Exchange

Our goal today is to have fun while we obtain more practice evaluating integrals using integration by substitution. We’ll introduce the idea of thinking of integration by substitution as the process of exchanging one integral (hard) for a different one (easy).

Group Work

You will split into groups of 3 or 4 students and be assigned one of the integrals below.

First: identify the \( u(x) \) function which will allow you to “exchange” the given integral for a simpler integral which you can evaluate easily.

Second: Convert your definite integral in \( x \)-variables to one in the new variables (usually \( u \)). Don’t forget the limits of integration!

Third: Evaluate your assigned integral by evaluating your definite integral in the new variables.

Each group will then write their solution on the board so the rest of the class can follow each solution. Each group will need someone with good handwriting to write-up the solution and another person who can explain the written solution and someone else who can answer questions about the solution. Then we will go through the solutions together. When we are done everyone should understand how to do all 8 integrals.

1. \[ \int_{2}^{4} 2x^2(x^3 - 3)^{12} \, dx = \]
2. \[ \int_{0}^{\pi/2} \cos(2x)\sqrt{\sin(2x)} \, dx = \]
3. $\int_{0}^{5} \sin(e^x)e^{x}+\cos(e^x)\,dx =$

4. $\int_{0}^{1} \frac{x}{2x + 1}\,dx =$

5. $\int_{0}^{4} \frac{2x^2 - 4}{(x^3 - 6x + 20)^3}\,dx =$

6. $\int_{1}^{8} e^{\sqrt[3]{x}} \frac{dx}{\sqrt[3]{x^2}} =$

7. $\int_{0}^{1} \sqrt{1 + 31x}\,dx =$

8. $\int_{1}^{2} x\sqrt{x - 1}\,dx =$