SHOW YOUR WORK

Consider the improper integrals below. Write down whether you think the integral converges or diverges in the box (1 point). By selecting an appropriate integral (1 point), prove your selected choice is correct by using the comparison theorem (3 points).

(a) (5 points) \( J = \int_1^\infty e^{2t+t+1} \, dt \) **DIVERGES**

We need to show \( J > a \) divergent integral.

We know \( \int e^t \, dt = \lim_{b \to \infty} \int_1^b e^t \, dt = \lim_{b \to \infty} (e^b - e^1) = \infty \)

\( t > 1 \)
\( t^2 > t \)
\( t^2 + t > t \)
\( t^2 + t + 1 > t \)
\( e^{t^2 + t + 1} > e^t \)
\( \int e^{t^2 + t + 1} \, dt > \int e^t \, dt \)

So \( J \) diverges by comparison to \( \int e^t \, dt \).

(b) (5 points) \( K = \int_1^\infty \frac{1}{\sqrt{s^4 + 1}} \, ds \) **CONVERGES**

As \( s \to \infty \), \( \frac{1}{\sqrt{s^4 + 1}} \sim \frac{1}{s^2} \), which we know \( \int_1^\infty \frac{1}{s^2} \, ds \) converges.

We need to show \( K < a \) convergent integral.

\( s > 1 \)
\( s^2 > 1 \)
\( s^4 > s^2 \)
\( s^4 + 1 > s^4 \)
\( \frac{1}{s^4 + 1} < \frac{1}{s^4} \)
\( \frac{1}{\sqrt{s^4 + 1}} \leq \frac{1}{\sqrt{s^4}} = \frac{1}{s^2} \)

So, by comparison, \( \int \frac{1}{s^2} \, ds \) converges for \( p = 2 > 1 \).