BONUS Quiz 2

Name: ____________________________  Prof. Ron Buckmire

Date: ________________  Friday February 28
Time Begun: ________________
Time Ended: ________________

Topic covered: Improper Integration

The student learning outcome of this quiz is for you to illustrate your understanding of using the comparison technique to determine convergence or divergence of improper integrals.

Reality Check:

EXPECTED SCORE : __________/10  ACTUAL SCORE : __________/10

Instructions:

1. Once you open the quiz, you have 30 minutes to complete it.

2. You may not use the book or any of your class notes, but you may use a calculator. You must work alone.

3. If you use extra paper, please staple it to the quiz before coming to class. UNSTAPLED SHEETS WILL NOT BE GRADED.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules. Complete the reality check to give yourself a sense of how well you think you did on the quiz.

5. Relax and enjoy....

6. This quiz is due on Monday, March 3, at the beginning of class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.
SHOW ALL YOUR WORK AND EXPLAIN EVERY ANSWER

Consider the function \( f(z) = \frac{1}{\sqrt{z} + z} = \frac{1}{z^{1/2} + z} \). The task for this quiz is to determine whether \( I = \int_{1}^{\infty} \frac{1}{\sqrt{z} + z} \, dz \) and \( J = \int_{0}^{1} \frac{1}{\sqrt{z} + z} \, dz \) converge or diverge without evaluating any integrals.

(a) (4 points) Use the rules developed in class to say whether each of the following integrals converges or diverges:

\[
\int_{1}^{\infty} \frac{1}{\sqrt{z} + z} \, dz \quad \int_{0}^{1} \frac{1}{\sqrt{z} + z} \, dz \quad \int_{0}^{\infty} \frac{1}{2\sqrt{z}} \, dz \quad \int_{1}^{\infty} \frac{1}{2z} \, dz
\]

(b) (1 point) For \( z > 1 \) is \( \frac{1}{\sqrt{z} + z} > \frac{1}{z + z} \) or \( \frac{1}{\sqrt{z} + z} < \frac{1}{z + z} \)? (CIRCLE YOUR ANSWER)

For \( z > 1 \) is \( \frac{1}{\sqrt{z} + z} > \frac{1}{\sqrt{z} + \sqrt{z}} \) or \( \frac{1}{\sqrt{z} + z} < \frac{1}{\sqrt{z} + \sqrt{z}} \)? (CIRCLE YOUR ANSWER)

(c) (2 points) Does \( I = \int_{1}^{\infty} \frac{1}{\sqrt{z} + z} \, dz \) converge or diverge? Support your answer by explaining how the inequalities in part (b) and your answers to part (a) allow you to determine whether \( I \) converges or diverges without actually evaluating it. If you like, you can use a graph to support your explanation.

(d) (1 point) For \( 0 < z < 1 \) is \( \frac{1}{\sqrt{z} + z} > \frac{1}{z} \) or \( \frac{1}{\sqrt{z} + z} < \frac{1}{z} \)? (CIRCLE YOUR ANSWER)

For \( 0 < z < 1 \) is \( \frac{1}{\sqrt{z} + z} > \frac{1}{\sqrt{z}} \) or \( \frac{1}{\sqrt{z} + z} < \frac{1}{\sqrt{z}} \)? (CIRCLE YOUR ANSWER)

(e) (2 points) Does \( J = \int_{0}^{1} \frac{1}{\sqrt{z} + z} \, dz \) converge or diverge? Support your answer by explaining how the inequalities in part (d) and your answers to part (a) allow you to determine whether \( J \) converges or diverges without actually evaluating it. If you like, you can use a graph to support your explanation.