HW 7
MATH 120

Sec 7.2: # 3, 7, 13, 32
Sec 7.3: # 2, 5, 11, 16

7.2.3
\[ x = \sqrt{4y} \quad 2\sqrt{y} = 1 \quad y = \frac{x^2}{4} \]

Area bounded by \( y = \frac{x^2}{q}, y = 9, x = 0 \) rotated about y-axis

Disk Method
\[ V = \int_0^9 \pi x^2 \, dy = \int_0^9 4\pi y \, dy = \pi \left[ 2y^2 \right]_0^9 = 162\pi \]

7.2.7
\[ y^2 = x \]

Washer Method
\[ V = \int_0^2 \pi \left( (2y)^2 - (y^2)^2 \right) \, dy = \pi \int_0^2 4y^2 - y^4 \, dy = \pi \left[ \frac{4y^3}{3} - \frac{y^5}{5} \right]_0^2 = \pi \left( \frac{4 \cdot 8 - 32}{5} \right) = \pi \cdot 32 \cdot \left( \frac{2}{15} \right) = \frac{64\pi}{15} \]
**7.2.13** \( y = \frac{1}{x}, x = 1, x = 2, y = 0 \) about \( x \)-axis

**Disk Method**

\[
V = \int_1^2 \pi y^2 \, dx = \pi \int_1^2 \left( \frac{1}{x} \right)^2 \, dx
\]

\[
= \pi \int_1^2 x^{-2} \, dx = \pi \left[ -\frac{1}{x} \right]_1^2
\]

\[
= \pi \left( -\frac{1}{2} - (-1) \right) = \pi \left( \frac{1}{2} \right)
\]

\[
= \frac{\pi}{2}
\]

**7.2.32**

**Using Similar Triangles**

\[
\frac{E}{H-h} = \frac{R}{H} \quad \Rightarrow \quad \frac{rH}{R} = \frac{R(H-h)}{H} = R(1 - \frac{rH}{R}) = \frac{R}{R-r}
\]

\[
\frac{R_h}{r} = \frac{H}{R-r}
\]

\[
V = \frac{1}{3} \pi \left[ \frac{R^2}{R-r} \cdot \frac{R_h}{R-r} - \frac{r^2}{R-r} \cdot \frac{R_h}{R-r} + \frac{R^2}{R-r} \cdot \frac{r^2}{R-r} \right]
\]

\[
= \frac{1}{3} \pi \left[ R^3 - r^2 (R^2 + Rr + r^2) \right]
\]

\[
= \frac{1}{3} \pi \left( R^3 - r^2 (R^2 + Rr + r^2) \right)
\]

\[
= \frac{1}{3} \pi \left( R^3 - r^2 \left( \frac{R}{R-r} \right) \left( \frac{R^2}{R-r} + Rr + r^2 \right) \right)
\]

\[
= \frac{1}{3} \pi \left( R^3 - r^2 \left( \frac{R^3}{R-r} \right) \right)
\]

\[
= \frac{1}{3} \pi \left( R^3 - r^2 \left( \frac{R^3}{R-r} \right) \right)
\]

\[
V = \frac{1}{3} \pi \left( R^3 + Rr + r^2 \right)
\]
Sec 7.3.2

\[ V = \int_{0}^{\sqrt{\pi}} 2\pi x \sin(x^2) \, dx = \pi \int_{0}^{\sqrt{\pi}} x \sin(x^2) \, dx \]

\[ V = \pi \int_{0}^{\sqrt{\pi}} \sin(u) \, du \]

\[ = -\pi \cos(u) \bigg|_{0}^{\pi} \]

\[ = -\pi [\cos(\pi) - \cos(0)] = -\pi [-1 - 1] = -\pi \cdot 2 = 2\pi \]

7.3.5.

\[ V = \int_{0}^{1} 2\pi xe^{-x^2} \, dx \]

\[ = \pi \left[ e^{-x^2} \right]_{0}^{1} \]

\[ = \pi \left( e^{-1} - 1 \right) = \pi \left( 1 - \frac{1}{e} \right) \]

7.3.11

\[ V = \int_{0}^{8} 2\pi y \left( \frac{y^2}{3} \right) dy = 2\pi \int_{0}^{8} \frac{y^3}{3} \, dy \]

\[ = 2\pi \left[ \frac{y^4}{12} \right]_{0}^{8} \]

\[ = 2\pi \left( \frac{8^4}{12} \right) = 2\pi \left( \frac{4096}{12} \right) = \frac{768\pi}{7} \]

\[ V = \int_{0}^{8} 2\pi y \left( \frac{y^2}{2} \right) dy = 2\pi \int_{0}^{8} \frac{y^3}{2} \, dy \]

\[ = 2\pi \left[ \frac{y^4}{8} \right]_{0}^{8} \]

\[ = 2\pi \left[ \frac{8^4}{8} - 0 \right] = 2\pi \left[ \frac{256}{8} - 0 \right] = 2\pi \left[ 32 - 0 \right] = 64\pi \]

\[ V = \int_{0}^{8} 2\pi y \left( \frac{y^2}{3} \right) dy = 2\pi \int_{0}^{8} \frac{y^3}{3} \, dy \]

\[ = 2\pi \left[ \frac{y^4}{12} \right]_{0}^{8} \]

\[ = 2\pi \left( \frac{8^4}{12} \right) = 2\pi \left( \frac{4096}{12} \right) = \frac{768\pi}{7} \]
\[ V = \int_0^1 2\pi (1 + x) \sqrt{x} \, dx \]

\[ = 2\pi \int_0^1 \left( x^{1/2} + x^{3/2} \right) \, dx \]

\[ = 2\pi \left[ \frac{2}{3} x^{3/2} + 2x^{5/2} \right]_0^1 \]

\[ = 2\pi \left( \frac{2}{3} + \frac{2}{5} \right) = 4\pi \left( \frac{2}{15} + \frac{2}{15} \right) \]

\[ = \frac{32\pi}{15} \]