15. \[ \int \frac{1 - \sin x}{\cos x} \, dx = \int \frac{\sec x - \sin x}{\cos x} \, dx = \int \sec x - \tan x \, dx = \ln |\sec x + \tan x| - \ln |\sec x| + C \]

Let \( u = \cos x \) \( du = -\sin x \, dx \) \[ \int \frac{1}{u} \frac{du}{(-\sin x)} + \int \frac{du}{u} = \int \frac{1}{u} \frac{-1}{\sqrt{1-u^2}} \, du + \ln |u| \]
\[ = \ln \left| \frac{1 + \sqrt{1-u^2}}{u} \right| + \ln |\cos x| \]
\[ = \ln \left| \frac{1}{\cos x} + \frac{\sin x}{\cos x} \right| - \ln |\sec x| + C \]
\[ = \ln |\sec x + \tan x| - \ln |\sec x| + C \]

37. (a) \( \cos(A + B) = \cos A \cos B - \sin A \sin B \)
\( \cos(A - B) = \cos A \cos B + \sin A \sin B \)
\( \cos(A - B) - \cos(A + B) = 2 \sin A \sin B \)
\( \frac{1}{2} [\cos(A - B) - \cos(A + B)] = \sin A \sin B \)
(b) \[ \int \sin 5x \sin 2x \, dx = \int \frac{\cos 3x - \cos 7x}{2} \, dx = \frac{1}{2} \left( \frac{\sin(3x) - \sin(7x)}{3} \right) + C \]
\[ = \frac{1}{6} \sin 3x - \frac{1}{14} \sin 7x + C \]
Sec 6.2 #63
\[
\int \sqrt{5 + 4x - x^2} \, dx = \int \sqrt{5 + 4 - (2 - x)^2} \, dx
\]
\[= \int \sqrt{3^2 - (2 - x)^2} \, dx\]
Looks like \[\int \sqrt{a^2 - u^2} \, du\] where \(a = 3\) and \(u = 2 - x\)

Table of Integrals #30
\[-\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a}\right) + C\]
\[-\frac{9}{2} \sqrt{5 + 4x - x^2} - \frac{9}{2} \sin^{-1}\left(\frac{2 - x}{\sqrt{3}}\right) + C\]
\[-\frac{x - 2}{2} \sqrt{5 + 4x - x^2} + \frac{9}{2} \sin^{-1}\left(\frac{x - 2}{\sqrt{3}}\right) + C\]

Sec 6.3 #8
\[
\int \frac{3t - 2}{t + 1} \, dt = \int 3 - \frac{5}{t + 1} \, dt
\]
\[= 3t - 5 \ln|t + 1| + C\]

Sec 6.3 #19
\[
\int \frac{x^2 + 1}{(x - 3)(x - 2)^2} \, dx
\]
\[= \int \frac{10}{x - 3} - \frac{9}{x - 2} - \frac{5}{(x - 2)^2} \, dx
\]
\[= 10 \ln|x - 2| - 9 \ln|x - 2| + 5 \cdot \frac{1}{x - 2} + C
\]

\[
1 + x^2 = A \cdot \frac{1}{x - 3} + B \cdot \frac{1}{x - 2} + C \cdot \frac{1}{(x - 2)^2}
\]

Let \(x = 2\) \(S = C - 1 \Rightarrow C = -5\)

Let \(x = 3\) \(10 = A - 1 \Rightarrow A = 10\)

Let \(x = 0\)
\[1 = A \cdot 4 + B(-3)(-2) + C(-3)
\]
\[1 = 4A + 6B - 3C
\]
\[1 = 55 + 6B
\]
\[-54 = 6B \Rightarrow B = -9
\]
Sec 6.4

#2 \[ \int_0^2 x^2 \sqrt{4-x^2} \, dx \]

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\[ \int u^2 \sqrt{a^2-u^2} \, du = \frac{u(2u^2-a^2)}{8} \sqrt{a^2-u^2} + \frac{a^4}{8} \sin^{-1}\left(\frac{u}{a}\right) + c \]

\[ a = 2 \]

\[ \int_0^2 x^2 \sqrt{2^2-x^2} \, dx = \left. \frac{x(2x^2-4)\sqrt{4-x^2}}{8} \right|_0^2 + \frac{2^4}{8} \sin^{-1}\left(\frac{2}{2}\right) \left|_0^2 \right. \]

\[ = 0 - 0 + 2 \sin^{-1}(1) - 2 \sin^{-1}(0) \]

\[ = 2 \cdot \frac{\pi}{2} = \pi \]

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\[ \int \cos^4 x \, dx = \int \cos^2 x \cdot \cos^2 x \, dx = \int \left(1 + \cos 2x \right)^2 \, dx \]

\[ \cos 2x = 2 \cos^2 x - 1 \]

\[ \frac{1 + \cos 2x}{2} \]

\[ = \frac{1}{4} \int \left(1 + \cos 2x + \frac{1 + \cos 4x}{2} \right) \, dx \]

\[ = \frac{3}{8} x + \frac{\cos 2x}{4} + \frac{\cos 4x}{8} \, dx \]

\[ = \frac{3}{8} x + \frac{\sin 2x}{8} + \frac{\sin 4x}{32} + C \]

Using Mathematica