Preparing for the exam

1. **The ideas are the most important thing! I believe you are well-prepared to succeed on this exam!**
2. **Problems will resemble quiz, homework and worksheet questions.** But they will not be identical to these. One good way to review is to examine each quiz and worksheet, deciding which major idea(s) each exercise is trying to illuminate. Another excellent way to study is to make up a mock exam. Make a list of the major themes and skills and write problems of three sorts: those which test one (or maybe two) basic skills, those which test for understanding of major themes and those which combine the two. Then give your mock exams to each other. It might be advantageous to make up problems which do not look anything like previous problems. I have given you a copy of questions I wrote for Calculus 2 exams from previous years.
3. **Make sure you understand central concepts.** Important concepts include:
   - **Infinity is a trend, NOT a number**! Taking the limit of $f(x)$ as x approaches c is NOT ALWAYS the same thing as "plugging in" the value c into the function $f(x)$, especially if the value c is **infinity**. You should remember that $\lim_{x\to\infty}$ means x is getting very very large and consider the corresponding behavior of the function. The Rules we developed in class for limits should help you evaluate limits. You should make up your own limit problems and try and do them. You can always make a table and set the trend for the input $\lim_{x\to\infty}$ to determine the trend as $f(x) \to f(\infty)$.

Limit Formulas

$$\int_a^\infty \frac{dx}{x^p} = \begin{cases} \textbf{DIVERGES} & \text{when } p \le 1 \\ \textbf{CONVERGES} & \text{when } p > 1 \end{cases}$$

$$\int_0^b \frac{dx}{x^q} = \begin{cases} \textbf{DIVERGES} & \text{when } q \ge 1 \\ \textbf{CONVERGES} & \text{when } q < 1 \end{cases}$$

$$\lim_{x\to\infty} e^{kx} = \begin{cases} 0 & \text{when } k < 0 \\ \infty & \text{when } k > 0 \end{cases}$$

$$\lim_{x\to\infty} x^r = \begin{cases} 0 & \text{when } r < 0 \\ \infty & \text{when } r > 0 \end{cases}$$

$$\lim_{x\to 0^+} x^r = \begin{cases} \infty & \text{when } r < 0 \\ 0 & \text{when } r > 0 \end{cases}$$

4. Other topics include:
   a. improper integrals of the first kind and of the second kind
   b. absolute convergence versus conditional convergence, absolute convergence implies convergence
   c. polynomial approximations of a function near a point (Taylor polynomials), applications of Taylor polynomial approximations to derivatives and anti-derivatives
   d. power series approximations of a function near a point (Taylor series), how to use calculus and algebra to find new Taylor series from familiar ones.
   e. tests for convergence of infinite series **n-th term, alternating series, integral, basic comparison, absolute ratio and limit comparison**
   f. useful series to remember are p-series, geometric series, harmonic series (it diverges!), alternating harmonic series (it converges!)
   g. Remember the differences between improper integrals, infinite series, infinite sequences

5. **Practice using tests for convergence.** Especially important are the Absolute Ratio Test and the n-th Term Test (i.e. Buckmire Divergence Test). Don't come into the exam without being able to take the limit as $k \to \infty$ of some expression involving k. You should be able to apply L'Hopital's rule on those indeterminate limits. Don't forget the other tests we have covered (the Integral Test, the Basic Comparison Test, Limit Comparison, Absolute Comparison and Alternating Series Test).

6. Remember the basic idea of doing comparisons:
   If you want to show that something CONVERGES, you have to compare it to something which is **LESS THAN OR EQUAL TO** something you already know CONVERGES.
   If you want to show that something DIVERGES, you have to compare it to something which is **GREATER THAN OR EQUAL TO** something you already know DIVERGES.
   The "something" can either be an improper integral or an infinite series, but in either case the integrand or terms must all be POSITIVE.

7. Taylor Series To Remember...

$$\begin{aligned}
\sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots &&= \sum_{k=0}^{\infty} \frac{(-1)^{k+1} x^{2k+1}}{(2k+1)!} \\
\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots &&= \sum_{k=0}^{\infty} \frac{(-1)^{k+1} x^{2k}}{(2k)!} \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \ldots &&= \sum_{k=0}^{\infty} x^k \\
(1+x)^n &= 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots &&= \sum_{k=0}^{\infty} \frac{n!}{k!(n-k)!} x^k \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots &&= \sum_{k=1}^{\infty} (-1)^{(k+1)} \frac{x^k}{k} \\
e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots &&= \sum_{k=0}^{\infty} \frac{x^k}{k!}
\end{aligned}$$

8. **Evaluating Limits**
   <u>L'Hôpital's Rule</u>
   If $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is of the form $\frac{\infty}{\infty}$ or $\frac{0}{0}$ or $0 \cdot \infty$ then
   if if the limit $\lim_{x\to\infty} \frac{f'(x)}{g'(x)} = L$ exists, then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ In other words, if you have an indeterminate limit, just differentiate the numerator and denominator and take the limit again until you get a determinate answer. That answer will be the value of the limit.
   You should be comfortable with discounting or ignoring parts of an expression when these parts are getting very small compared to the rest of the expression. Remember, $\sin(x)$ and $\cos(x)$ only return values between ± 1.

   <u>Exotic Indeterminate Limits</u>
   $\lim_{x\to\infty} f(x)^{g(x)} = \lim_{x\to\infty} e^{g(x)\ln(f(x))} = e^{\lim_{x\to\infty} g(x)\ln(f(x))}$.

9. **Interval Of Convergence and Radius Of Convergence**
   Consider $\sum_{k=0}^{\infty} b_k (x-a)^k$. This Power Series may not converge for all x-values. The set of x-values for which the series converges is called the *interval of convergence*. The interval of convergence is always centered on the point a.
   The interval of convergence can be infinite, i.e. $(-\infty, \infty)$ a.k.a. "all Real Numbers". Or it can be a finite interval of the form $(a-R, a+R)$, $[a-R, a+R]$, $(a-R, a+R]$ or $[a-R, a+R)$. R is called the *radius of convergence*. The value $R = 1/L$ and is computed using the absolute ratio test.

$$\frac{1}{R} = L = \lim_{k\to\infty} \left| \frac{b_{k+1}}{b_k} \right|$$

   Recall that if a power series is formed by taking the derivative or anti-derivative of another power series than the resultant power series has the **same** *radius of convergence* but may have a different *interval of convergence*. (Examples of this are the power series for $\frac{1}{1+x^2}$ and $\arctan(x)$)

Rules for the Exam

1. **BLUE NOTES:** You are allowed the attached half-sheet of “blue” paper for written notes. Only the use of notes on this blue sheet of paper will be permitted during the exam. **You may not use your calculator or your cellphone during the exam.** This policy will, of course, be reflected in test questions. There will be fewer problems involving simple calculations and more involving conceptual understanding and explanation of ideas.
2. You must take the exam during your regularly scheduled lab time unless you have made prior arrangements with me.
3. As usual, there will be a pledge on the exam. By signing the pledge, you indicate that you followed all the rules of this exam and furthermore that you promise not to discuss the exam with anyone (even people who have already finished the exam) until after **4:30 pm on Thursday, April 17, 2014**. It is our collective responsibility to keep the exam as fair as possible.
4. No answer will be given credit without accompanying work. No exceptions. Unless otherwise indicated, answers should be left in exact form, i.e. no decimal approximations. I can not stress strongly enough that you must write your solutions in an **intelligible** and **coherent** fashion. When you are writing a solution to a problem you are attempting to communicate with the reader (me) how you solved the problem. I can assure you that if I do not how you arrived at an answer, YOU WILL NOT GET CREDIT FOR THAT ANSWER. The correct answer is important, yes, but assuring me that you know what technique needs to be used to arrive at this answer is more important. I would be very happy to see full sentences written explaining your answers.
5. This list of rules is not necessarily exhaustive. If you have any questions about what is allowed and what is not, you are responsible for asking me. Ignorance is not an excuse.

BLUE NOTES

Name: ______________________________

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