Report on Exam 3

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Point Distribution (N=21)

Range	92.5+	90+	87.5+	82.5+	80+	77.5+	72.5+	70+	67.5+	62.5+	60+	60-
Grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	8	3	2	2	1	1	1	1	1	0	0	0

Summary The results on Exam 3 were the best of any of the in-class exams! A record 11 of 21 students earned an A (or A-) with the even better news that no students failed the exam. The median score was extremely high (90) while the mean score was 87. The high score was 101. Congratulations to everyone who did as well as they hoped!

#1 Convergence Tests for Infinite Series. ANALYTIC, COMPUTATIONAL. **(a)** $\sum_{k=1}^{\infty} \ln(k)$ DIVERGES by the n^{th} term test. $\lim_{k\to\infty} \ln(k) = \infty \neq 0$. **(b.)** $\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$. DIVERGES by Integral Test. Or Diverges by Basic Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k}$. **(c.)** $\sum_{k=1}^{\infty} \frac{\ln(k)}{k^3}$ CONVERGES by Integral Test or by Basic Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$. **(d.)** $\sum_{k=1}^{\infty} (-1)^k \frac{k^2}{k^2+2k+1}$. DIVERGES by Alternating Series Test (it fails part 1, since $\lim_{k\to\infty} \frac{k^2}{k^2+2k+1} = 1 \neq 0$). **(e.)** $\sum_{k=1}^{\infty} \frac{1}{k^2+2k+1}$ CONVERGES by Integral Test. Since $\frac{1}{k^2+2k+1} = \frac{1}{(k+1)^2}$ so it is easy to inegrate. Or it CONVERGES by Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$. **(f.)** $\sum_{k=1}^{\infty} (-1)^k 2^{-k}$ CONVERGES by Geometric Series test or Absolute Ratio Test since it is a geometric series with $r = -1/2$ which is less than one in magnitude.

#2 Sequences, Series, Convergence. ANALYTIC, VERBAL, CREATIVE. **MADISON** is the student who demonstrates the most understanding of Calculus. Both Sydney and Terry think that the alternating harmonic series diverges. The main point here is to write at least four truthful sentences about the ideas expressed by Madison, Sydney and Terry in order to get full credit.

#3 ANALYTIC, VERBAL, COMPUTATIONAL, CREATIVE. **Limits, Sequences, L'Hopital's Rule**. Most people were able to come up with a function $a(k)$ that has an infinite limit when $k \to \infty$. The simplest choice is $a(k) = e^k$. Also most people could think of a function which looks like its limit is indeterminate but is actually zero. The simplest choice for that is $b(k) = \frac{k}{e^k}$. The point of part (c) is that your functions are SEQUENCES not SERIES. So, convergence means which one has a finite limit when $k \to \infty$. That would be $b(k)$. $a(k)$ is a divergent sequence, but $b(k)$ is a convergent sequence. There's no reason to talk about convergence tests, since those are used for series and this problem is about sequences.

#4 Power Series, Interval of Convergence, Radius of Convergence. ANALYTIC, COMPUTATIONAL, VISUAL. The power series is $\sum_{k=1}^{\infty} \frac{2^k}{k} x^k$. You show it has a radius of $\frac{1}{2}$ by taking the limit of $\frac{1}{R} = \lim_{k\to\infty} \left|\frac{c_{k+1}}{c_k}\right| = \lim_{k\to\infty} \frac{2^{k+1}\cdot k}{2^k\cdot(k+1)} \lim_{k\to\infty} \frac{2k}{k+1} = 2$ So you know the values of x between $-1/2$ and $1/2$ will converge. You need to check the endpoints. If you plug in $x = -1/2$ you get the alternating harmonic series, which CONVERGES (so this point is included). If you plug in $x = 1/2$ you get the harmonic series which diverges (so this point is excluded). Thus the actual interval of convergence is $-1/2 \leq x < 1/2$.

BONUS $M(x) = \sum_{k=1}^{\infty} \frac{2^k}{k} x^k$. By looking closely, you can see that this can be rewritten as $\sum_{k=1}^{\infty} \frac{(2x)^k}{k}$

So, basically this power series looks like $\sum_{k=1}^{\infty} \frac{(\square)^k}{k}$. By scanning your Blue Notes, you should see that

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{(k+1)} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

therefore

$$\begin{aligned} \ln(1-x) &= \sum_{k=1}^{\infty} (-1)^{(k+1)} \frac{(-x)^k}{k} = -x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \dots \\ &= \sum_{k=1}^{\infty} (-1)^{(k+1)} (-1)^k \frac{(x)^k}{k} \\ &= \sum_{k=1}^{\infty} (-1)^{(2k+1)} \frac{x^k}{k} \end{aligned}$$

$$-\ln(1-x) = \sum_{k=1}^{\infty} \frac{x^k}{k} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

So this means that $-\ln(1-\square) = \sum_{k=1}^{\infty} \frac{\square^k}{k} = \square + \frac{\square^2}{2} + \frac{\square^3}{3} + \frac{\square^4}{4} + \dots$

which means that $-\ln(1-2x) = \sum_{k=1}^{\infty} \frac{(2x)^k}{k}$

So the Mystery Function $M(x) = -\ln(1-2x)$ for x-values in $[-1/2, 1/2)$.