PRACTICE FINAL EXAM
MATH 120

14. Taylor Series for $f(x) = e^{-x^3}$ about the point $x = -1$.

YOU HAVE TO TAKE DERIVATIVES

$$T_n(x) = f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \dots$$

$f(x) = e^{-x^3}$ $\quad f(-1) = e^{-(-1)^3} = e^{--1} = e^1$

$f'(x) = e^{-x^3} \cdot -3x^2$ $\quad f'(-1) = e^1 \cdot -3(-1)^2 = -3e$

$f''(x) = e^{-x^3} \cdot (-3x^2)^2 + e^{-x^3} \cdot -6x$ $\quad f''(-1) = e^1 \cdot 9 + e \cdot 6 = 15e$

$$T_n(x) = e - 3e(x+1) + \frac{15}{2}(x+1)^2 + \dots$$

The question probably SHOULD have been

Find Taylor Series for $f(x) = e^{-x^3}$ about $x = 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \frac{x^k}{k!} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$e^{-x^3} = 1 - x^3 + \frac{(-x^3)^2}{2!} + \frac{(-x^3)^3}{3!} + \dots \frac{(-x^3)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{3k}}{k!}$$

$$= 1 - x^3 + \frac{x^6}{2!} - \frac{x^9}{3!} + \frac{x^{12}}{4!} - \dots$$

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15. Find radius & interval of convergence of $\sum_{k=0}^{\infty} (-4)^k x^{2k+1}$

Absolute Ratio Test

$$L = \lim_{k\to\infty} \left| \frac{(-4)^{k+1} x^{2(k+1)+1}}{(-4)^k x^{2k+1}} \right| = \lim_{k\to\infty} \left| \frac{(-4) x^{2k+3}}{x^{2k+1}} \right|$$

$$= \lim_{k\to\infty} |-4x^2| = |-4x^2|$$

$$L = 4x^2 < 1$$

For convergence $L < 1$ so $4x^2 < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

So the radius of convergence is $\frac{1}{2}$

$-\frac{1}{2}$ 0 $\frac{1}{2}$

We need to check the endpoints to find the interval of convergence

At $x = \frac{1}{2}$

$$\sum_{k=0}^{\infty} (-4)^k \left(\frac{1}{2}\right)^{2k+1} = \sum_{k=0}^{\infty} (-4)^k \left(\frac{1}{2}\right)^{2k} \cdot \frac{1}{2}$$

$$= \sum_{k=0}^{\infty} (-1)^k (2^2)^k \frac{1}{2^{2k}} \cdot \frac{1}{2}$$

$$= \sum_{k=0}^{\infty} (-1)^k \cdot \frac{1}{2}$$ DIVERGES by nth term test

At $x = -\frac{1}{2}$

$$\sum_{k=0}^{\infty} (-4)^k \left(-\frac{1}{2}\right)^{2k+1} = \sum_{k=0}^{\infty} (-1)^k (-1)^{2k+1} \left(\frac{1}{2}\right)^{2k} \frac{1}{2} 4^k$$

$$= \sum_{k=0}^{\infty} (-1)^{k+1} \cdot \frac{1}{2}$$ DIVERGES by nth term test

$$\lim_{k\to\infty} (-1)^k \frac{1}{2} = DNE \neq 0$$

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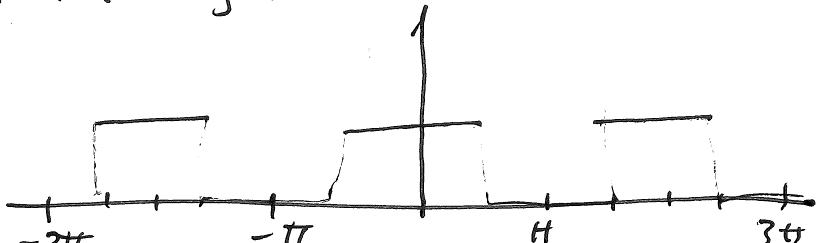
16. $f(x) = \begin{cases} 0, & -\pi < x < -\pi/2 \\ 1, & -\pi/2 < x < \pi/2 \\ 0, & \pi/2 < x < \pi \end{cases}$

Sketch the graph of $f(x)$ period is 2π



$f(-x) = f(x)$
symmetric about y-axis
so its even

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\,dx = \frac{1}{2\pi}\, x \Big|_{-\pi/2}^{\pi/2} = \frac{1}{2\pi}\left(\frac{\pi}{2}\right) - \left(\frac{-\pi}{2}\right) = \frac{\pi}{2\pi} = \frac{1}{2}$$

$$= \frac{\text{area under graph}}{2\pi} = \frac{\pi \cdot 1}{2\pi} = \frac{1}{2}$$

We know all $b_k = 0$ because the function is even so no SINE coefficients

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} 1 \cdot \cos kx\,dx$$

$$= \frac{\sin kx}{k\pi}\Big|_{-\pi/2}^{\pi/2} = \frac{\sin\frac{k\pi}{2} - \sin\left(-\frac{k\pi}{2}\right)}{k\pi}$$

$$a_k = \frac{2}{k\pi}\sin\frac{k\pi}{2}$$

$$F_\infty = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{2}{k\pi}\sin\left(\frac{k\pi}{2}\right)\cos kx = \frac{1}{2} + \frac{2}{\pi}\cos x - \frac{2}{3\pi}\cos 3x + \frac{2}{5\pi}\cos 5x - \frac{2}{7\pi}\cos 7x + \ldots$$

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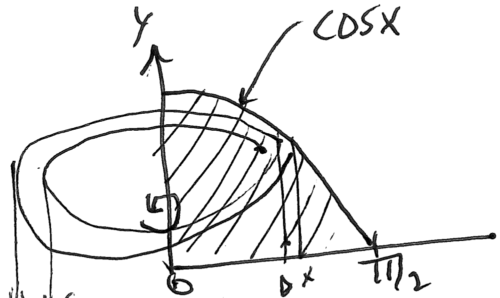
Stewart p422, #17

$$\int_0^{\pi/2} 2\pi x \cos x \, dx \leftarrow$$ This uses method of shells.

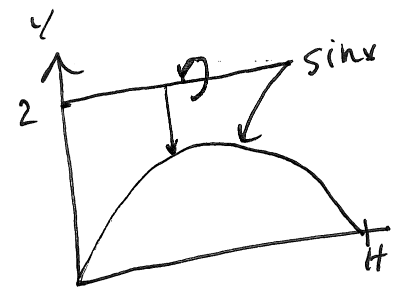
This is the volume formed by rotating the area bounded by $\cos x$ between $x=0$ and $x=\pi/2$ and the x-axis AROUND the y-axis



422, #19

$$\int_0^{\pi} \pi(2-\sin(x))^2 \, dx \leftarrow$$ This used method of washers

This is the volume formed by rotating the $y=\sin x$ curve between $x=0$ and $x=\pi$ about the $y=2$ line



422, #20

SHELLS

$$\int_0^4 2\pi(6-y)(4y-y^2)\,dy$$

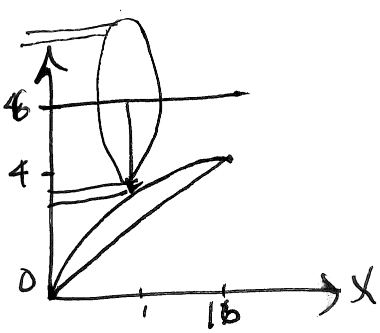
2π · RADIUS · HEIGHT

This is the volume formed by rotating the area $4y-y^2$ between $y=0$ & $y=4$ around the $y=6$ axis.

$x=4y \Rightarrow y=\frac{x}{4}$

$x=y^2$

This is the same area as between $y=\sqrt{x}$ and $y=\frac{x}{4}$ on $x=0$ to $x=16$



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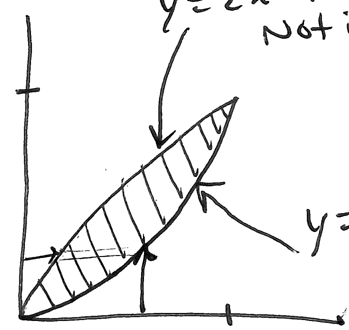
Stewart, pg 422, #14

$y = x^3$ $\qquad x^3 = 2x - x^2$

$y = 2x - x^2 \qquad x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x-1)(x+2) = 0$



(a) Area of R $= \int_0^1 2x - x^2 - x^3 \, dx$

$= \left(x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 1 - \frac{1}{3} - \frac{1}{4} = \frac{12}{12} - \frac{4}{12} - \frac{3}{12} = \boxed{\frac{5}{12}}$

(b) Rotate about the x-axis

$V = \int_0^1 \pi (2x - x^2)^2 - (x^3)^2 \, dx \quad = \cancel{\int_0^1 2\pi x}$ can't use shells! because $y = 2x - x^2$ can't be inverted

Using Washers ↑

$= \pi \int_0^1 4x^2 - 4x^3 + x^4 - x^6 \, dx = \pi\left(\frac{4x^3}{3} - x^4 + \frac{x^5}{5} - \frac{x^7}{7}\right)\Big|_0^1$

$= \pi\left[\frac{4}{3} - 1 + \frac{1}{5} - \frac{1}{7}\right] = \pi\left[\frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right] = \pi\left[\frac{35 + 21 - 15}{105}\right]$

$= \frac{41}{105}\pi$

(c) Rotate about the y-axis

can't use washers because you can't find $x = f(y)$

$V = \int_0^1 2\pi x \,(2x - x^2 - x^3)\, dx$

Using shells

$= 2\pi \int_0^1 2x^2 - x^3 - x^4 \, dx$

$= 2\pi\left(\frac{2x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1$

$= 2\pi\left(\frac{2}{3} - \frac{1}{4} - \frac{1}{5}\right) = 2\pi\left(\frac{40 - 15 - 12}{60}\right) = \frac{26\pi}{60}$

$= \frac{13\pi}{30}$