PARTIAL FRACTIONS

CLASS #9

1 $\frac{2x^2-x+4}{x^3+4x} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$ $\quad A=1, B=1, C=0$

2. $\frac{10}{5x^2-2x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{5-2x}$ $\quad C=\frac{8}{5}$, $B=2$, $A=\frac{4}{5}$

3. $\frac{x^6}{x^2-4} = x^4+4x^2 + \frac{A}{x+2} + \frac{B}{x-2}$ $\quad A=-4, B=4$

4. $\frac{x^5+1}{(x^2-x)(x^4+2x^2+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$

$x^5+1 = A(x^2+1)^2(x-1) + B(x^2+1)^2 x + (Cx+D)x(x-1)(x^2+1) + (Ex+F)(x)(x-1)$

$x=1 \quad 2 = B\cdot 2^2 \Rightarrow \boxed{B=1/2}$

$x=0 \quad 1 = A\cdot -1 \Rightarrow \boxed{A=-1}$

$x=-1 \quad 0 = A\cdot 2^2\cdot(-2) + B\cdot 2^2\cdot -1 + (D-C)(-1)(-2)(2) \Rightarrow \frac{3}{2}=C$

$\mathcal{O}(x^5) \quad 1 = A+B+C \Leftrightarrow C = 1-A-B = 1-(-1)-\frac{1}{2} = \frac{3}{2}$

$C-D = -2A-B = -2\cdot(-1)-\frac{1}{2} = \frac{3}{2}$

$\boxed{D=0}$

$\mathcal{O}(x^0) \quad 1 = -A-D \Rightarrow \boxed{D=0}$

$\mathcal{O}(x^1) \quad 0 = A+B+D+F \Rightarrow \boxed{F=1/2}$

let $x=2$: $33 = A\,25\cdot 1 + B\cdot 25\cdot 2 + (2C+D)2(1)\cdot 5 + (5E+F)\cdot 2\cdot 1$

$33 = A\cdot -1\cdot 25 + \tfrac{1}{2}\cdot 25\cdot 2 \quad 33 = 30+1+10E \quad \boxed{E=1/5}$