The Method of Separation of Variables
Class 21: Monday March 24

Initial Value Problems

Recall that an *initial value problem* consists of a **rate equation** and an **initial condition**. For example,

$$y' = f(x), \qquad y(a) = b$$

The solution of an initial value problem is a specific FUNCTION $y(x)$ about which we know its DERIVATIVE (i.e. $y' = f(x)$) and one value it goes through (i.e. when $x = a, y = b$).

1. Consider the function $\mathcal{A}(x) = b + \int_a^x f(t)dt$ Does it solve the initial value problem? PROVE IT!

GroupWork

2. Write down the general solution to: $y' = \sec^2(x), \qquad y(0) = -1$

3. The non-integral form of the solution is, therefor:

4. Write down the solution of the following differential equation:
$y' = \sin(x) - \cos(x) + e^{2x}$

5. How many solutions did you find?

6. How many solutions are there to the initial value problem below? Find them. $y' = \sin(x) - \cos(x) + e^{2x}, \qquad y(0) = 2$

7. So, in general (i.e. for any $f(t)$) we can write down the solution of **any** initial value problem:
$y' = f(t), \qquad y(a) = b$

Separation of Variables
We can also solve differential equations of the form

$$y' = f(x)g(y), \quad y(a) = b$$

by using a technique called Separation of Variables.
We can re-write y', the derivative of $y(x)$, as $\dfrac{dy}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= f(x)g(y) \\ \frac{dy}{g(y)} &= f(x)dx \\ \int \frac{dy}{g(y)} &= \int f(x)dx \end{aligned}$$

Example
8. Let's solve the following initial value problem

$$y' = ky, \qquad y(0) = A$$

GroupWork
9. Use Separation of Variables to solve the following initial value problem

$$\frac{dy}{dt} = yt, \qquad y(0) = 2$$