

Applications of Definite Integrals
Class 17: Friday March 7

Goal

Increase the number of different kinds of integrals we can evaluate through more practice, and learn some applications.

Sometimes you have to use external information (usually other mathematical facts) in order to evaluate an integral:

1. $\int \sin^2(x) dx =$

2. $\int \frac{1}{x^2}(x^3 - 4) dx =$

3. $\int \ln(x^4) dx =$

4. $\int \sqrt{x}(x - 1)^2 dx$

Length of a curve segment

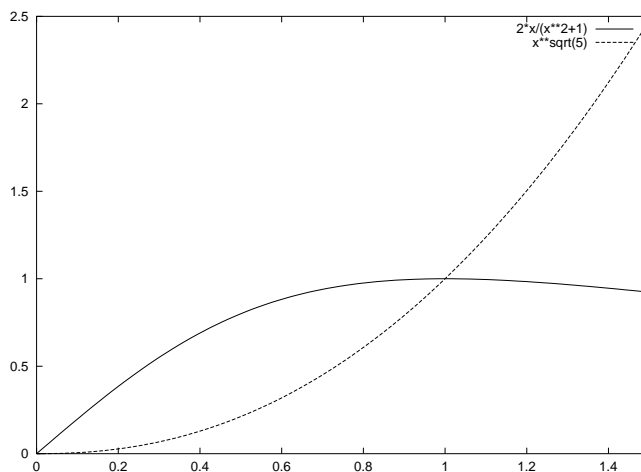
Previously we have learned that one application of an integral is that the length of a curve segment between two points $(a, f(a))$ and $(b, f(b))$ is given by

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Area between two curves

Look at the figure below and write down an integral which represents the value of the shaded area

A. The area A represents the area between two curves, $f(x) = x\sqrt{5}$ and $g(x) = \frac{2x}{1+x^2}$



Compute the value of A by evaluating the definite integral:

So, in general the area between two curves $f(x)$ and $g(x)$, with $f(x) > g(x)$ on $[a, b]$ is given by:

Average value of a function

What is the average value of the natural log function between 1 and 4?

You should be able to write down an integral which represents this number and answer the question by evaluating this integral.

The average value, \bar{f} , of a function $f(x)$ on some interval $a \leq x \leq b$ on which it varies continuously is given by the integral: