

Understanding the Fundamental Theorem of Calculus
Class 10: Wednesday February 12

Let's make sure we understand the Fundamental Theorem of Calculus by considering the following conversation between two Calculus students about the best way to evaluate the integral

$$\int_0^{2\pi} \sin(x)dx.$$

Madison: There's no way to get the exact value of $\int_0^{2\pi} \sin(x)dx$. I know we can always estimate the value of a definite integral using a Riemann Sum so we should do that in this case with $N = 1$ and any sample point to obtain a fairly good estimate of the value of the integral.

Sydney: Riemann Sums are useless! Instead, we can always use the Fundamental Theorem of Calculus to evaluate a definite integral. All we have to do is plug in $x = 2\pi$ into $\sin(x)$ and then plug in $x = 0$ into $\sin(x)$, and subtract the difference. The exact value of the integral is therefore $\sin(2\pi) - \sin(0) = 0$.

Madison: Well, you have to do all that work to get an approximate answer and I can just plug some symbols into any TruBasic program, set $N = 1000000$ and also get a really really accurate estimate, so I think my way is better!

Discuss the student's understanding of the Fundamental Theorem of Calculus, carefully identifying any false statements. Do not make any false statements yourself! Proofread your answer.

Look at the table of derivatives and anti-derivatives below.
What operation do you have to do to move from the left to the right?

What operation do you have to do to move from the right to the left?

Fill in as much of the table as you can.

$\int f(x) dx$	$f(x)$	$\frac{d}{dx} (f(x))$
	1	
	x	
	$x^n (n \neq -1)$	
	$\frac{1}{x}$	
	$\sin(x)$	
	$\cos(x)$	
	e^x	
	$a^x (a > 0)$	
	$g'(x)$	