Connecting Riemann Sums to the Definite Integral
Class 5: Friday January 31

Warm-up Our goal today is to make that eveyone fully understands the connections between Riemann Sums and Definite Integrals. We will also start to think about Accumulation Functions. To that end, take 3 minutes and complete the following sentences.

1. A **Riemann Sum** can be written mathematically as ...

2. A **Definite Integral** can be computed from a **Riemann Sum** by ...

AREA as a Riemann Sum
Let's look at a simple area problem for which we know we can find the answer *exactly*. 3. What is the area under the curve $f(x) = 3x$ from the origin ($x = 0$) to some point $x = L$? (HINT: what is the shape that forms?)

4. Sketch a picture of this area in the figure below. Then we'll try and compute the answer using a Riemann Sum.

We will use a Riemann Sum using an equipartition on [0,L] with N subintervals to approximate the area. In order to do this you will need to answer a few questions:
5. What is Δx for your partition? (NOTE: An equipartition is what we call the set of points you get when you split up an interval into a number of equal parts.)

6. In general, to find Δx on an equipartition of n subintervals on $[a, b]$ the formula is...

7. Now write down a **Right-hand Riemann Sum** to approximate the shaded area.

8. What is a formula for the **exact area** under the curve, using the Riemann Sum?
(How do estimates made using Riemann Sums become more accurate?)

We can also represent the exact value of the area by the symbol $\int_0^L 3x\,dx$.
This is called a **definite integral**.

PAIR WORK

Write down a sentence translating the mathematical symbols $\int_a^b f(x)\,dx$ into English and then compare your sentence with your neighbor's.

DISTANCE TRAVELLED as a Riemann Sum

Consider the graph below of **velocity versus time**, $v(t)$ in (kilometers per hour) versus t (in hours) below. The graph represents the motion of a 747 in its flight from LAX to JFK.

9. How would you compute the distance travelled D from time $t = 1$ to $t = 6$ using a Riemann Sum?

10. Is there a mathematical way that you could write an expression which would represent the *exact* value of the distance travelled? Write it now.

Now think about the connections about these two problems.
11. What **quantity** needs to be "summed" in the Riemann Sum when you are computing distance? In other words, what is *accumulated* to produce your area estimate?

DISTANCE TRAVELLED is the accumulation of __________________________ with _______________
Riemann sum looks like:

AREA is the accumulation of __________________________ with ____________________.
Riemann sum looks like:

Try this one... **VOLUME** is the accumulation of __________________________ with _________________
Riemann sum looks like:

Thus, given a function we can define a related **Accumulation Function** which can be approximated using Riemann Sums and defined exactly using an integral.

Given $f(x)$, an **accumulation function for** $f(x)$ is written as $A(\mathcal{X})$, where $A(\mathcal{X}) = \int_a^{\mathcal{X}} f(x)dx$