Math 120 Spring 2001 — FINAL EXAM

Basic Calculus 2

Ron Buckmire
Alan Knoerr

Name: ______________________________

Section (8:30am or 10:30am): ____________________

Instructions:

1. There are six (**6**) questions on this exam. Questions may involve computations or interpretations or both. Read and answer each question carefully and fully. The clearer you explain exactly what you are doing to solve each problem, the easier it will be to give you the credit you so richly deserve.
2. This exam is written to be completed in one and a half hours, but you have the full three (**3**) hour final exam period to finish it.
3. Partial credit will be given, but only if we can see the correct parts of the solution. **Show all of your work.**
4. Recall the ground rules attached to your blank blue notes sheet. Only your blue notes are allowed. Your blue notes must be handed in with your completed exam. When you are finished you must sign the pledge below.
5. Take a deep breath. Relax and enjoy the exam. You should begin by at least reading through every problem. If you have any questions about the problems let us know.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score
1	/25
2	/25
3	/25
4	/25
5	/25
6	/25
Total	**/150**

1. **Fundamental Theorem Of Calculus**.

   a. (10 points). Write down an example of an integral which CAN be evaluated using the fundamental theorem of calculus, and **evaluate** it.

   b. (10 points). Write down an example of an integral which CAN NOT be evaluated using the fundamental theorem of calculus and **explain** why you can't evaluate it exactly.

   c. (5 points). Use your example from part (b.) and state a technique you would use to *estimate* the value of this integral. Discuss how you would improve the accuracy of your estimate.

2. **Indefinite Integrals/Techniques of Integration.**

   Compute the following antiderivatives.

   (a) (8 points). $\int 5 + y^5 + \cos(5y - 5)\ dy$

   (b) (8 points). $\int \ln(5t) + \sqrt[5]{t + 55}\ dt$

   (c) (9 points). $\int x^5 \ln(x)\ dx$

3. **Numerical Integration.**

Two students are discussing different approaches to evaluating a particular integral, $I = \int_0^4 e^{\sqrt{x}} dx$

Madison: Clearly there is no way to evaluate this integral.

Tyler: But this is a definite integral with limits of integration, so we must be able to apply the Fundamental Theorem of Calculus to obtain a value for it (it's just a number).

Madison: Hmmm, well now that you mention that it's a definite integral and not just an antiderivative, I remembered that we can estimate any definite integral using Simpson's Rule.

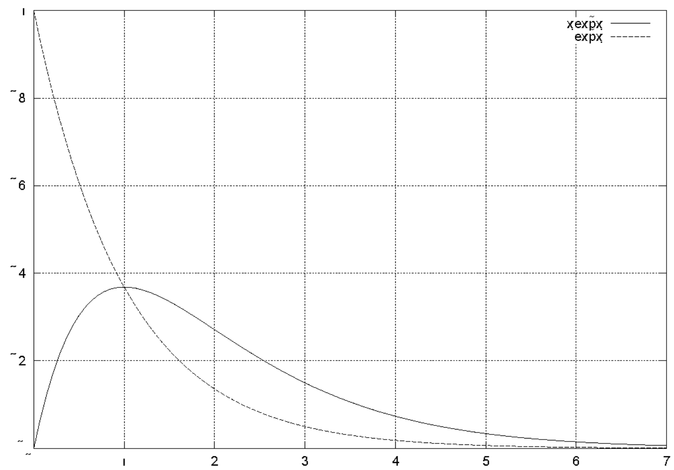
Tyler: Why use Simpson's Rule to evaluate I? I much prefer the Midpoint Riemann Sum method with $\Delta x = 4$, i.e. $N = 1$. For this particular integral I know that I will get an overestimate of $4e^{\sqrt{2}}$ for the actual value of the integral.

Madison: Why use $N = 1$? I would go to `TruBasic` and use $N = 100$ with `RIEMANN.TRU` and get an estimate which is about 10 000 times more accurate than yours!

Write a couple of paragraphs discussing the students' understanding of numerical techniques for evaluating integrals. Explain why each statement is correct or incorrect. **You must be careful not to make any incorrect statements yourself in your explanation.** PROOFREAD YOUR ANSWER.

4. **Applications of Integration.**

Consider the shaded region A bounded by the functions $f(x) = e^{-x}$, $g(x) = xe^{-x}$ and the **entire** positive x-axis.



a. Find the exact value of the area of the shaded region A

5. **Infinite Series/Improper Integrals**

Use an appropriate test for convergence or divergence to determine if each of the following expressions converges or diverges. Be sure to state the test you use and justify your use of that test.

a. (5 points). $\sum_{k=0}^{\infty}(-1)^k \frac{k^2}{k^2+2k+1}$

b. (5 points). $\sum_{k=0}^{\infty}(-1)^k 2^{-k}$

c. (5 points). $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$

d. (5 points). $\int_1^{\infty} te^{-t^2}\,dt$

e. (5 points). $\int_0^1 \frac{1}{\sqrt{1-x}}dx$

6. **Taylor Series**

a. (15 points). Use the Taylor series for $\ln(1+x)$ at $a = 0$, $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}$ to show that the Taylor series for $\ln(1-x^2)$ at $a = 0$ is $\sum_{k=1}^{\infty} -\frac{x^{2k}}{k}$

b. (10 points). Use the absolute ratio test to find the interval of convergence of $\sum_{k=1}^{\infty} -\frac{x^{2k}}{k}$, i.e. the x-values for which the Taylor series for $\ln(1-x^2)$ will converge.