Math 120 Spring 2003

FINAL EXAM

Basic Calculus 2

Ron Buckmire

Name: ______________________________

Section (10:30am or 2:30pm): ____________________

Instructions:

1. There are six (**6**) questions on this exam. Questions may involve computations or interpretations or both. Read and answer each question carefully and fully. The clearer you explain exactly what you are doing to solve each problem, the easier it will be to give you the credit you so richly deserve.
2. This exam is written to be completed in one and a half hours, but you have the full three (**3**) hour final exam period to finish it.
3. Partial credit will be given, but only if we can see the correct parts of the solution. **Show all of your work.**
4. Recall the ground rules attached to your blank blue notes sheet. Only your blue notes are allowed. Your blue notes must be handed in with your completed exam. When you are finished you must sign the pledge below.
5. Take a deep breath. Relax and enjoy the exam. You should begin by at least reading through every problem. If you have any questions about the problems let us know.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score
1	/25
2	/25
3	/25
4	/25
5	/25
6	/25
Total	**/150**

1. **Infinite Series**.

   a. (10 points). Write down an example of an infinite series which **CONVERGES** and apply an appropriate test to PROVE it converges.

   b. (10 points). Write down an example of an infinite series which **DIVERGES** and apply an appropriate test to PROVE it diverges.

   c. (5 points). Write down an example of a nonzero infinite series which converges to a known value, and give that value.

2. **Indefinite Integrals/Techniques of Integration.**

Compute the following antiderivatives.

(a) (8 points). $\int 2^2 + y^2 + 2^y \, dy$

(b) (8 points). $\int 2^{r^2} 2r + r^2 2r \, dr$

(c) (9 points). $\int 2^t 2t \, dt$

3. **Initial Value Problems.**

a. Find the function $y(x)$ which solves the following initial value problem. CHECK YOUR ANSWER!

$$\frac{dy}{dx} = xe^x, \quad y(0) = 1$$

b. Find the function $y(x)$ which solves the following initial value problem. CHECK YOUR ANSWER!

$$\frac{dy}{dx} = ye^x, \quad y(0) = 1$$

4. **Applications of Integration: Fourier Series**

Suppose $f(x)$ is a periodic function with period 2π, defined by:

$$f(x) = \begin{cases} 0 & \text{if} \quad -\pi < x \leq -\pi/2 \\ 2\pi & \text{if} \quad -\pi/2 < x \leq \pi/2 \\ 0 & \text{if} \quad \pi/2 < x \leq \pi \end{cases}$$

a. (5 points). Sketch the function $f(x)$ in the space below, on the interval $-3\pi \leq x \leq 3\pi$

b. (20 points). Find the third-degree Fourier polynomial, $F_3(x)$. [HINT: Is the function $f(x)$ even or odd or neither?]

5. **Numerical Integration**

Consider the improper integral

$$A = \int_0^1 x^x \, dx$$

a. (5 points). Use a LEFT hand Riemann Sum with N=1 to obtain an estimate of A

b. (5 points). Use a RIGHT hand Riemann Sum with N=1 to obtain an estimate of A

c. (5 points). Use a MIDPOINT Riemann Sum with N=1 to obtain an estimate of A

d. (5 points). Use the TRAPEZOID method with N=1 to obtain an estimate of A

e. (5 points). Use SIMPSON'S RULE with N=1 to obtain an estimate of A

6. **Taylor Series**

a. (15 points). Show that the Taylor Polynomial of degree 2 for the function $f(x) = x^x$ about the point $a = 1$ is

$$P_{2,a=1}(x) = 1 + (x-1) + (x-1)^2.$$

[NOTE: $f'(x) = x^x(\ln(x) + 1)$]

b. (10 points). Use the Taylor Polynomial of degree 2 for the function $f(x) = x^x$ about $a = 1$ from part (a) to obtain an approximate value for

$$A = \int_0^1 x^x dx$$