Math 120 Spring 2000

FINAL EXAM

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Name: ____

Section (8:30am or 9:30am): ______________________

Instructions:

1. There are eight (**8**) questions on this exam. Four in Part 1 and four in Part 2. You need to choose three problems for each part. Each problem is worth 20 points for a total of 120 points.

   Questions may involve computations or interpretations or both. Read and answer each question carefully and fully. The clearer you explain exactly what you are doing to solve each problem, the easier it will be to give you the credit you so richly deserve.

2. This exam is written to be completed in an hour and a half, but you have the full three (**3**) hour final exam period to finish it.

3. Partial credit will be given, but only if we can see the correct parts of the solution. **Show all of your work.**

4. Only your blue notes are allowed. Your blue notes must be handed in with your completed exam. When you are finished you must sign the pledge below.

5. Take a deep breath. Relax and enjoy the exam. You should begin by at least reading through every problem. If you have any questions about the problems let us know.

Pledge: I, ______________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Part I.	Score:
1	/20
2	/20
3	/20
4	/20
Part II.	Score:
5	/20
6	/20
7	/20
8	/20
Total:	/120

Part 1

1. (a) Find $\displaystyle\int_{-\infty}^{-1} \frac{2x}{(1+x^2)^2}\, dx$

   (b) Verify that $\displaystyle\int_{-\infty}^{\infty} \frac{2x}{(1+x^2)^2}\, dx$ is zero.

2. (a) Find the Taylor series for the funtion $f(x) = \ln(x)$ centered at $a = 1$.

(b) Find the interval of convergence for the Taylor Series you found in part (a).

(c) What is the Power series for $g(x) = x\ln(x) - x$ at $a = 1$?

3. (a) In this part, you will make numerical approximations of $\int_0^1 \sqrt{1+e^{2x}}\,dx$ based ONLY on the table of function values below:

x	$(1+e^{2x})^{(1/2)}$
0	1.4142
0.1	1.4904
0.2	1.5786
0.3	1.6799
0.4	1.7960
0.5	1.9283
0.6	2.0785
0.7	2.2484
0.8	2.4399
0.9	2.6551
1	2.8964

Pick on appropriate number of subintervals n, and compute the left-hand Riemann sum L, the right-hand Riemann sum R, the midpoint Riemann sum M, the Trapezoid rule T and the Simpson's rule approximation S. Use the same number of subintervals for each of these approximations. USE ONLY THE VALUES IN THE TABLE AND USE AS MUCH ACCURACY AS YOUR CALCULATOR WILL ALLOW. DO NOT ROUND OFF TO FEWER THAN 4 DECIMAL PLACES.

$L =$

$R =$

$M =$

$T =$

$S =$

(b) Arrage L, R, M, T and S in order from the smallest underestimate to largest overestimate for this function. Explain, in complete sentences, your reasons for placing them in the order you select and why, for this function, this order is true for any value of n (any choice of number of subintervals). (Note: You do not need all of the inequality signs below.)

$$\leq \qquad \leq \qquad \leq \qquad \leq \text{ Actual Value } \leq \qquad \leq \qquad \leq \qquad \leq$$

(c) Explain why the definite integral $\int_0^1 \sqrt{1+e^{2x}}\, dx$ equals the arclength of the curve $y = e^x$ between the points $(0, 1)$ and $(1, e)$.

4. Evaluate the limits in parts (a) and (b) using l'Hopital's rule and then answer the question in part (c).

a. $\lim_{k \to \infty} \frac{\sin\left(\frac{1}{k}\right)}{\frac{1}{k}}$

b. $\lim_{k \to \infty} \frac{\sin\left(\frac{1}{k^2}\right)}{\frac{1}{k^2}}$

c. *Don't Panic!* Another test for convergence of a positive infinite series (which we did not discuss in class but which is very useful) is the **LIMIT COMPARISON TEST**. Here it is, in a nutshell.

Say we *want* to know if the positive series $\sum_{k=1}^{\infty} a_k$ converges or diverges and we *know* what the positive series $\sum_{k=1}^{\infty} b_k$ does (either converges or diverges), then if we can show that

$$\lim_{k\to\infty} \frac{a_k}{b_k} = L,$$

where $0 \leq L < \infty$, we may conclude that the series $\sum_{k=1}^{\infty} a_k$ behaves exactly in the same way $\sum_{k=1}^{\infty} b_k$. In other words, as long as the limit above exists, if $\sum_{k=1}^{\infty} b_k$ converges, so does $\sum_{k=1}^{\infty} a_k$; if $\sum_{k=1}^{\infty} b_k$ diverges, then so does $\sum_{k=1}^{\infty} a_k$.

For example, we can use this test to determine if the series $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$ and $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{1}{k^2}\right)$ converge or diverge by comparing them to $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$, respectively. Based on your work in parts (a) and (b) above, explain why the series $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$ and $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{1}{k^2}\right)$ either converge or diverge.