Basic Calculus 2

Name: _____

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1. There are ten (10) questions on this final exam. You are to choose three (3) out of the first five, and three (3) out of the last 5, for a total of **six (6)** problems you are supposed to do.
2. Read each question you choose carefully and be sure to answer appropriately. When it makes sense, answer in complete sentences.
3. Partial credit will be given, but only if we can see the correct parts. So **show all of your work**.
4. You have 3 hours to complete this exam. The exam has been designed to be completed in 2 hours.
5. When you are finished please sign the pledge below. Remember it is our *collective* responsibilty to maintain a high standard of academic honesty.
6. Relax and enjoy...

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. We would like to find out whether the improper integral $\int_1^\infty \frac{\ln(x)}{x^2} dx$ converges or diverges.

(a) First, show that if $u = \ln(x)$ and $e^u = x$ then $\int_1^\infty \frac{\ln(x)}{x^2}\, dx = \int_0^\infty ue^{-u}\, du$.

(b) Use integration by parts to evaluate $\int_0^\infty ue^{-u}\, du$ and thus answer the question whether $\int_1^\infty \frac{\ln(x)}{x^2}\, dx$ converges or diverges.

2. Determine whether each of the following series converges or diverges. State clearly which test(s) you are using to support your answer.

(a) $\sum_{k=1}^{\infty} \ln(k)$

(b) $\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$

(c) $\sum_{k=1}^{\infty} \frac{\ln(k)}{k^2}$

3. Let $f(x) = x^2$, and $g(x) = 1 - x^2$.

(a) Sketch, on the same axes, graphs of f and g. Find the exact coordinates of the points where the two graphs intersect.

(b) Find the area between the two graphs.

4. (a) Use the formula $f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2! + \cdots$ to find the fourth degree Taylor polynomial for $f(x) = \sin(x)$ near $a = 0$.
(Note: you must show how you use the above formula.)

(b) Use your answer above to approximate $\sin(0.1)$. Then use your calculator to find the error in this approximation.

5. Suppose $f(x)$ is a periodic function with period 2π, defined by:

$$f(x) = \begin{cases} 1 & \text{if } -\pi < x \leq 0 \\ -1 & \text{if } 0 < x \leq \pi \end{cases}$$

Find the second degree Fourier approximation to this function.

6. Evaluate the following integrals.

(a) $\int x^2 + t^2 + x\sin(t^2) + x\sin(x^2)\ dx$

(b) $\int_1^2 \sqrt{7x+2}\ dx$

(c) $\int x\sqrt{7x+2}\ dx$

7. (a) Use integration by parts to show that $\int \ln(x)\, dx = x\ln(x) - x + C$.

(b) Find all antiderivatives of the function $f(x) = \sin^3(x^5)\cos(x^5)x^4$

8. Ice is forming on a pond at a rate given by

$$\frac{dy}{dt} = 0.5t^{1/2}$$

where y is the thickness of the ice in inches at time t measured in hours since the ice started forming. (So at $t = 0$ the thickness of the ice was zero.)

(a) Find y as a function of t.

(b) Verify that your answer above is indeed a solution to the given IVP.

9. (a) Use Simpson's Rule with $N = 2$ subintervals to approximate the definite integral

$$\int_{-1}^{1} 1 - x^2 \, dx$$

(b) What is the error in the above approximation? Explain.

10. (a) Does the harmonic series, $\sum_{k=1}^{\infty} \frac{1}{k}$, converge or diverge? Prove your answer.

(b) Does the alternating harmonic series, $\sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$, converge or diverge? Prove your answer.