Report on Exam 3
Point Distribution (N=39)

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<th>60-</th>
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</thead>
<tbody>
<tr>
<td>Grade</td>
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<td>A-</td>
<td>B+</td>
<td>B</td>
<td>B-</td>
<td>C+</td>
<td>C</td>
<td>C-</td>
<td>D</td>
<td>F</td>
</tr>
<tr>
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<td>2</td>
<td>3</td>
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</tbody>
</table>

Comments

Overall A much better performance from the class overall! The average score was the highest of the semester, an 83, up from 71 last time and beating the 82 on Exam 1. There were 5 people who got perfect scores of 100!

#1 This was a very straightforward problem where I was trying to see you apply as many convergence tests as possible. **10:30 version:** you can use Non-Zero Limit, Integral, Absolute Ratio, Root, and Comparison Tests. \( \sum_{k=0}^{\infty} e^k \) is also a geometric series. **2:30 version:** you can use the Non-Zero Limit, Integral and Comparison Tests. Your series was \( \sum_{k=1}^{\infty} 1 \) which can also be considered a p-series with \( p = 0 \) or a geometric series with \( a = r = 1 \). The Root test and Ratio tests will give you know information!

#2 This was a classic separation of variables problem. Some people either couldn’t remember or didn’t SHOW how they obtained the anti-derivative of \( \ln(x) \), which is \( x \ln x - x + C \). Everything in math is cumulative. You should have that anti-derivatives table on every blue notes sheet you have. Once you anti-differentiate you must have a constant of integration, which you can find out must be equal to zero if your solution \( y(x) \) is going to be the one particular member of the family of functions which obeys the differential equation AND the initial condition simultaneously. To confirm (i.e. check) an answer to an IVP you need to make sure the given \( y(x) = x^x \) obeys the D.E. and the I.C. In order to be able to differentiate \( x^x \) you must treat it as \( e^{\ln x} \).

#3 Before the exam I told you that you would need to be able to answer “constructive” questions where you construct an example of a series, limit or IVP which has the required properties, and we did several examples of these kinds of questions in the formal review sessions as well in the six hours of extended office hours the days before the exam.

(a) The basic idea of trying to find a limit which equals 4 is either to find a ratio of functions which after using L’Hopital’s Rule will get you 4, or find a limit which is equal to zero and add 4 to that function. **10:30** had to use the **exponential** function. **2:30** had to use the **natural logarithm** function. Both of these function go to infinity as \( k \to \infty \), so the functions \( 4 + 1/e^k \) or \( 4 + 1/\ln(k) \) will work, as will \( \frac{dx}{x+5} \) or \( 4 + \ln(1) \).

(b)

#4 This is very similar to Problem #3 except know you have to think of improper integrals of the first kind. The way to do this is to think about improper integrals you know which converge or diverge (OR infinite series which converge or diverge) and then add the appropriate function. One good technique is just to add the functions as **CONSTANTS**, i.e. \( e^0 = 1 \) or \( \ln(1) = 0 \) or \( \ln(e) = 1 \).

Using your p-test you know \( \int_1^{\infty} \frac{dx}{x^p} \) converges when \( p > 1 \). So the integral \( \int_1^{\infty} \frac{e^0}{x^2}dx \) CONVERGES and \( \int_1^{\infty} \frac{\ln(e)}{x} \) DIVERGES.