Math 120 Spring 2003

Report on Test 2

Ron Buckmire

Point Distribution (N=38)

Range	90+	85+	80+	74+	66+	59+	47+	42+	32+	32-
Grade	A	A-	B+	B	B-	C+	C	C-	D	F
Frequency	11	5	0	5	1	3	9	2	1	1

Comments

Overall Performance on this exam was not as excellent as on the first exam. The class average fell from 82 to 71. Again Section 2 (230pm: 76) had a higher average than Section 1 (1030am: 67). There were 2 perfect scores of 100 and a 99 and a 98. Nearly half (42%) of the class was able to earn either an A or A-. The median score was 75 and the mode was 49.
The exam was specifically designed to look *exactly* like the practice exam (Spring 2001) while testing for the techniques of integration by substitution (IBS) and integration by parts (IBP). The other topics in Unit 2 **will** be on the Final Exam.

#1 **(a)** This problem was just a list of four anti-derivative questions. The first one caused the most difficulty: finding the anti-derivative of $f(x) = \frac{1+x^2}{2x}$. Many people immediately tried IBS (won't work) or IBS (will work, but has to be done twice) instead of first SIMPLIFYING the integrand! Clearly, $\frac{1+x^2}{2x} = \frac{1}{2x} + \frac{x^2}{2x} = \frac{1}{2}\frac{1}{x} + \frac{x}{2}$ and you know the anti-derivative of $\frac{1}{x}$ and $\frac{x}{2}$. **(b)** The function $g(x) = \frac{2x}{1+x^2}$ is obviously related to $f(x)$ from part (a) but the integration methods are completely different. For $g(x)$ you can simply use IBS and obtain the answer $G(x) = \ln(1+x^2)$. **(c)** The function $h(x) = x\ln(x) - x$ should look familiar since it is the anti-derivative of $\ln(x)$. However, that means that the DERIVATIVE of $x\ln x - x$ is equal to $\ln(x)$. This question asks you for the ANTI-DERIVATIVE of $x\ln(x) - x$ which means that you need to use IBP to evaluate $\int x\ln(x) - x\ dx = \int x(\ln(x) - 1)$ (choose $u = \ln(x) - 1$ and $v' = x$) **(d)** This problem is very similar to part (b) in that the function involved looks like $2x \cdot \text{FUNCTION}(x^2)$ where this time the FUNCTION is $\cos(u)$ while in part (b) it was $1/u$. In addition this question asks about the anti-derivative of a "disguised constant" (e^2 in the 230pm version and $\sin(\pi/2)$ in the 1030am version). Of course the anti-derivative of a constant C is simply Cx.

#2 This problem is nearly identical to Problem 4 from Spring 1999 from the Practice Exam problems. It basically involves showing the technique of combining integration by substitution and integration by parts to evaluate a definite integral. Part (a) involves the integral $I = \int_0^1 te^t\,dt$ which must be done by IBP. Of course the value of a definite integral must be a number, which turns out to be 1. Part (b) GIVES you the $u(t)$ function to use in IBS which means you need to find du, change the limits from t-variables to u-variables and completely convert the original integral. Part (c) asks you to evaluate the new integral, which is $J = \int_1^e \ln(u)du = u\ln(u) - u|_1^e = e\ln e - e - (1\ln 1 - 1) = 1$. Even without knowing the value of J since it is obtained using IBS it MUST be the same value as I.

#3 This problem was very similar to the tables in Lab 3 and Problem 5 from Spring 2001. Basically all the parts are related and the questions on the bottom of the table are the most general form of the same derivatives and anti-derivatives above it. Thus you should notice that by choosing the correct value of P and Q on line 4 you can obtain all the functions on line 0 ($P=1, Q=0$), line 1 ($P=2, Q=0$), line 2 ($P=2, Q=3$) and line 3 ($P=5, Q=4$). The derivative of $\frac{1}{Px+Q}$ is $\frac{-1}{(Px+Q)^2} \cdot P$ and its anti-derivative is $\frac{1}{P}\ln(Px+Q)$. Line 5 is equal to line 4 except the function is multiplied by a constant, so the anti-derivative of $\frac{-1}{(Ax+B)^2} \cdot A \cdot A$ is $\frac{A}{Ax+B}$ whose antiderivative is $\ln(Ax+B)$.