

Occidental College Department of Mathematics  
Gateway – Derivatives  
Help Sheet

This sheet is to provide you with further information as you work toward achieving 90% proficiency on this gateway about **derivatives**. As you look through the key ideas below, try to create a realistic picture of what you understand and what you don't — the first attempt at the gateway should help you with this. While preparing for the second attempt, you should take full advantage of working with your peers, seeking help from other students (both in this course and others), seeing the peer counselors and professional staff at the Center for Teaching and Learning, and talking with your professors.

You need to know how to take the derivatives of the following types of functions, or how to use the following rules to find derivatives. Examples given are functions you should be able to differentiate.

1.  $y = a^x$ . The derivative of  $a^x$  is  $\ln a \cdot a^x$  for any constant  $a$ . Obviously the easiest case is  $a = e$  because then  $\ln e = 1$ . Ex:  $y = 5^x$

2. **Chain Rule.**

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

Ex:  $y = (3x^2 + 1)^3$

3. **Chain Rule – Multiple Times.** Generalize the above. So for the combination of three functions:

$$\frac{d}{dx}f(g(h(x))) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

Ex:  $y = \cos^2(3x + x^2)$

4.  $a^{f(x)}$ . This is a combination of 1. and 2. above and its derivative is  $\ln a \cdot a^{f(x)} \cdot f'(x)$ . Ex:  $y = e^{\cos(x)}$

5. **Basic functions and constants.** Remember that the derivative of any constant, even “weird-looking” ones like  $e$ ,  $\pi$ ,  $\ln 2$ , is always zero. Throughout this gateway, you also must know the derivative of all basic functions – polynomials, exponentials, natural log function, trig functions. Ex:  $y = \cos(x) + \pi$

6. **Quotient Rule.**

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Ex:  $y = \frac{\cos(x)}{x^3}$

7. **Chain Rule, Product Rule, Quotient Rule.** In this problem you must combine at least two of these rules. Ex:  $y = \sin(xe^x)$

8. **Natural Log.** Recall that the derivative of  $\ln x$  is  $\frac{1}{x}$ . However you may also have to utilize the chain rule if there is a function of  $x$  inside the natural log function. Ex:  $y = \ln(\cos(x))$

9. **Product Rule.**

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

Ex:  $y = \cos(x)\ln(x)$

10. **Polynomials.** Ex:  $y = x^{1994} - x^2 + 400$