Math 120, Spring2001 Exam #3

6:30pm, Wednesday April 25, 2001

Name: ____________________________

Section: 8:30am or 10:30am (circle one)

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1. There are four (4) questions on this exam. Each one involves both computations and interpretation. Read and answer each question carefully and fully. **Answers should be in complete sentences.**
2. The exam is scheduled to take 60 minutes (1 hour) but you have the full 3 hours to complete it.
3. Partial credit will be given, but only if we can see the correct parts. So **show all of your work**.
4. Recall the rules set out on the handout. Only your blue notes are allowed. Your blue notes must be handed in with your exam. When you are finished please sign the pledge below.
5. Relax and enjoy...and ask questions!

Pledge: I, ____________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Problem:	Score:
1a	/5
1b	/5
1c	/5
1d	/5
1e	/5
1f	/5
	/30
2a	/5
2b	/5
	/20
3a	/10
3b	/10
3c	/10
	/30
4	**/20**
TOTAL:	/100

1. (30 points) Determine if the following infinite series converge or diverge. In each case, state which test you use and show how you apply the test.

   a. $\sum_{k=2}^{\infty} (-1)^k \frac{1}{\ln(k)}$

   b. $\sum_{k=0}^{\infty} \sin(\frac{k\pi}{2})$

   c. $\sum_{k=1}^{\infty} \frac{k+1}{k}$

d. $\sum_{k=0}^{\infty} e^{-k}$

e. $\sum_{k=0}^{\infty} (-e)^k$

f. $\sum_{k=1}^{\infty} k^{-e}$

2. (20 points)

   a. Write down an example of an improper integral of the first kind which CONVERGES, and **explain** how you know your example is CONVERGENT.

   b. Write down an example of an improper integral of the first kind which DIVERGES, and **explain** how you know your example is DIVERGENT.

3. (30 points) Consider the integral $I_p = \int_1^e (\ln x)^p \frac{1}{x}\, dx$. We want to develop a rule for what values of p will I_p converge. Remember, p can be any real number. You should be able to find I_p regardless of what p actually is.

a. Apply the method of integration by substitution to I_p with $u = \ln(x)$. Show that the integral I_p can be written completely in terms of u as $\int_0^1 u^p\, du$.

b. Evaluate the integral from part (a) to determine for what values of p I_p converges and for what values of p it diverges.

c. Evaluate $I_3 = \int_1^e (\ln x)^3 \frac{1}{x}\, dx$

4. (20 points) Two students are discussing calculus and you overhear their conversation.

Sydney: The zero-limit test is the best test for infinite series! I just proved that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ converges because I know $\lim_{k\to\infty} \frac{1}{k} = 0$

Madison: That's not right! You should use the comparison test. Show that $\frac{1}{k}$ is greater than 1 for all $k > 1$. Then since we know $\frac{1}{k}$ is positive for all $k > 1$ and since $\sum_{k=1}^{\infty} 1$ DIVERGES, this will prove that the harmonic series is greater than a divergent series, and thus also diverges.

Comment on the understanding of calculus displayed by the two students. In clear, legible sentences identify any correct and incorrect statements made by the students. If a statement is incorrect explain why. **You must be careful not to make any incorrect statements yourself in your explanation.** PROOFREAD YOUR ANSWER.