Math 120 Spring2001 Exam #1

6:30pm, Thursday, February 22, 2001

Name: ____________________

Section (8:30am or 10:30am): __________

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1. There are five (5) questions on this exam. Each one involves both computations and interpretation. Read and answer each question carefully and fully. **Answers should be in complete sentences.**
2. The exam is scheduled to take 60 minutes (1 hour) but you have the full 3 hours to complete it.
3. Partial credit will be given, but only if we can see the correct parts. So **show all of your work**.
4. Recall the rules set out on the handout. Only your blue notes are allowed. Your blue notes must be handed in with your exam. When you are finished please sign the pledge below.
5. Relax and enjoy...and ask questions!

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Problem:	Score:
1a	/5
1b	/5
1c	/5
1d	/5
2	/20
3a	/10
3b	/10
4a	/7
4b	/7
4c	/6
5	/20
TOTAL:	/100

1. (*20 points*) We want to compare approximating the definite integral $D = \int_{-4}^{2} x^3 + 1\, dx$ using different Riemann Sums with $\Delta x = 2$.

(a) Using 3 rectangles and LEFT ENDPOINT sample points approximate the definite integral D.

(b) Using 3 rectangles and RIGHT ENDPOINT sample points approximate the definite integral D.

(c) Use the Fundamental Theorem of Calculus to evaluate the definite integral D exactly.

(d) Arrange the LEFT endpoint estimate L, the RIGHT endpoint estimate R and the exact value D in increasing order. (i.e. $L < D < R$, $L < R < D$, $D < L < R$, $D < R < L$, $R < D < L$ or $R < L < D$).

Explain why this result is reasonable given the way $f(x) = x^3 + 1$ behaves for $-4 \leq x \leq 2$.

2. (*20 points*) Consider the irregular shaped area shaded on the axes below. It is formed from the intersection of the curves $f_1(x) = 4x$, $f_2(x) = -4x$ and $f_3(x) = 10 - \frac{1}{2}x^2$.

Compute the size of the shaded area **A**, *exactly*. Explain in clear detail precisely how you compute the size of the shaded area, including what definite integrals and antiderivatives you had to compute.
NOTE: In order to assure yourself you have calculated correctly, *check that your value for* **A** *falls between an underestimate and overestimate for the shaded area.*

3. (*20 points*) Use the appropriate version of the Fundamental Theorem of Calculus to evaluate the following integral and find the following derivative. Show all of your work.

a. (*10 points*) $\int_1^2 \frac{2}{y^4} + 4^y \ dy =$

b. (*10 points*) $\frac{d}{dx} \int_{-2}^{x} \cos(t^2) \ dt =$

4. (*20 points*) Given the following information about an unknown function $g(x)$ and its antiderivative $G(x) = \int_1^x g(t)dt$

$$\int_1^2 g(u)\ du = 3, \quad \int_1^4 g(u)\ du = 15, \quad g(1) = 2, \quad g(2) = 4, \quad g(4) = 8$$

$$g'(1) = g'(2) = g'(4) = 2, \quad G(1) = 0, \quad G(2) = 3, \quad G(4) = 15$$

Use the Fundamental Theorem of Calculus to find the values of the numbers I, J and K.

(a) (*7 points*) Evaluate $I = \int_2^4 g(x)\, dx$

(b) (*7 points*) Evaluate $J = G'(2)$

(c) (*6 points*) Evaluate $K = \int_2^4 g'(x)\, dx$

5. (*25 points*) Two students are discussing calculus and you overhear their conversation.

Tyler: This is so cool! We can find the length of a curve $f(x)$ between the point $(a, f(a))$ and $(b, f(b))$ by just computing the value of a special definite integral. To be specific, you must evaluate $\int_a^b \sqrt{1-[f(x)]^2}\, dx$.

Madison: I don't believe you! Everyone knows that definite integrals can be estimated really, really accurately using Riemann sums, and that they only represent areas, so I know that we could never use a Riemann sum or a definite integral to compute a length.

Comment on the understanding of calculus displayed by the two students. In clear, legible sentences identify any correct and incorrect statements made by the students. If a statement is incorrect explain why. **You must be careful not to make any incorrect statements yourself in your explanation.** PROOFREAD YOUR ANSWER.