

We now have a stable of convergence tests at our disposal:

Zero-Limit Divergence Test

Absolute Ratio Test

Root Test

Alternating Series Test

Integral Test

Comparison Test

For each of the above tests write down a sentence or mathematical list of symbols which indicate your understanding of how they work. You may also want to indicate which ones you find are more useful than others.

While trying to determine convergence or divergence we need to take a lot of “tricky” limits.

Example 1

Suppose we try and use the Root Test to determine whether $\sum_{k=1}^{\infty} \frac{1}{k}$ converges. We'll need to evaluate

$$L = \lim_{k \rightarrow \infty} \left(\frac{1}{k}\right)^{1/k}$$

What can we say about $\ln(L)$?

Therefore we know that $L =$ _____

Example 2

Let's try and use the Ratio Test to show that $\sum_{k=1}^{\infty} \frac{1}{k^k}$ converges.

Example 3

$$\lim_{x \rightarrow 0^+} \ln(x) + \frac{1}{x}$$

Example 4

$$\lim_{x \rightarrow \infty} \frac{e^x}{x^{100}}$$