Class 27: Wednesday November 5

Introduction to Taylor Polynomials

Recall that we can use **The Microscope Equation** to approximate any differentiable function $f(x)$ near the point $x = a$ using the expression:

$$\Delta f \approx f'(a) \cdot \Delta x$$

which is also written as

$$f(x) \approx f(a) + f'(a)(x - a)$$

It is this property which allows us to write $\sin(x) \approx x$ and $\cos(x) \approx 1$ near $x = 0$.
Do you see how you can use the Miscroscope Equation to get these results?

$\sin(x) \approx \sin(0) + \cos(0)(x - 0)$

$\cos(x) \approx \cos(0) + -\sin(0)(x - 0)$

What is the **meaning** of the Microscope equation, graphically or visually?

Taylor's Theorem allows us to approximate a continuous, differentiable function $f(x)$ by a polynomial, and not just by a line. The definition of a Taylor Polynomial of degree n for $f(x)$ about a point $x = a$ is given by:

$$P_n(x; a) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 + \frac{f'''(a)}{6}(x - a)^3 + \ldots \frac{f^{(n)}}{n!}(x - a)^n = \sum_{k=0}^{n} \frac{f^{(k)}}{k!}(x - a)^k$$

Examples
Find the Taylor Polynomial of degree 7 for $f(x) = \cos(x)$ and $g(x) = \sin(x)$ about $x = 0$.
Note what information you need to find a Taylor Polynomial:

1. $\sin(x) \approx$

2. $\cos(x) \approx$

Notice any patterns?

Some Taylor polynomials are fun to compute. For example, we can write down the n-th degree Taylor Polynomial for $f(x) = e^x$ about $x = 0$ below:

3. $e^x \approx$

Another interesting one is the n-th degree Taylor Polynomial for $f(x) = \dfrac{1}{1-x}$ about $x = 0$

4. $\dfrac{1}{1-x} \approx$

patterns of Taylor Polynomials When computing Taylor Polynomials it helps to notice patterns so we don't have to differentiate our lives away...

Consider the function $\dfrac{1}{1+x}$. How is it different from **4.** above?

What does your intuition tell you it's Taylor Polynomial will look like...?

5. $\dfrac{1}{1+x} \approx$

<u>GroupWork</u>

Try to write the general Taylor Polynomial (of degree n, about $a = 0$) for the following functions by seeing a relationship between the given function and a function you already know the Taylor Polynomial for. (You can always check your answer the hard way....)

6. $\dfrac{1}{(1-x)^2} \approx$

7. $e^{x^2} \approx$

8. $e^{5x} \approx$

9. $\ln(1+x) \approx$