

**Definition of the Definite Integral**

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1. A **Riemann Sum** can be written mathematically as ...
2. A **Definite Integral** can be computed from a **Riemann Sum** by ...

**AREA as a Riemann Sum**

Let's look at a simple area problem for which we know we can find the answer *exactly*. 3. What is the area under the curve  $f(x) = 3x$  from the origin ( $x = 0$ ) to some point  $x = L$ ? (HINT: what is the shape that area forms?)

4. Sketch a picture of this area in the figure below. Then we'll try and compute the answer using a Riemann Sum.

We will use a Riemann Sum using an equipartition on  $[0, L]$  with  $N$  subintervals to approximate the area. In order to do this you will need to answer a few questions:

5. What is  $\Delta x$  for your partition? (NOTE: An equipartition is what we call the set of points you get when you split up an interval into a number of equal parts.)

6. In general, to find  $\Delta x$  on an equipartition of  $n$  subintervals on  $[a, b]$  the formula is...

7. Now write down a **Right-hand Riemann Sum** to approximate the shaded area.

8. What is a formula for the **exact area** under the curve, using the Riemann Sum?  
(How do estimates made using Riemann Sums with  $N$  subdivisions become more accurate?)