Class 14: Wednesday October 1

Multivariable Optimization

Critical Points for Functions of Two Variables

Definition: $z = f(x, y)$ has a **local maximum** at (x_0, y_0) if $f(x_0, y_0) \geq f(x, y)$, for all (x, y) in some neighborhood of (x_0, y_0).

Similarly, $z = f(x, y)$ has a **local minimum** at (x_0, y_0) if $f(x_0, y_0) \leq f(x, y)$, for all (x, y) in some neighborhood of (x_0, y_0).

1. Suppose we take a vertical slice in the x-direction through $f(x, y)$ at a local maximum (x_0, y_0), and suppose f is locally linear there. What will this slice look like near this point? What is the value of $f_x(x_0, y_0)$?

2. Suppose we take a vertical slice in the y-direction through $f(x, y)$ at a local maximum (x_0, y_0), and suppose f is locally linear there. What will this slice look like near this point? What is the value of $f_y(x_0, y_0)$?

3. What does the tangent plane to f at a locally linear local maximum (x_0, y_0) look like?

Definition: Suppose $f(x, y)$ is locally linear in a neighborhood of (x_0, y_0). Then (x_0, y_0) is a **critical point** for f if

$$f_x(x_0, y_0) = 0 \quad \text{and} \quad f_y(x_0, y_0) = 0.$$

Critical points are candidates for local maxima and local minima. Critical points which are not local maxima or local minima are called **saddle points**. Contour plots can help in classifying critical points.

4. Find critical points for the following functions:

a) $f(x, y) = -x^2 + y^2$

b) $g(x, y) = (x-1)^2 + (y-1)^2 + 3$

c) $h(x, y) = -(x/2)^2 - (y/3)^2$

Match these functions to the contour plots below, and classify the critical points as local maxima, local minima, or saddle points.