Math 118 **ADVANCED PLACEMENT CALCULUS** Fall 2003

Class 13: Monday October 29

Multivariable Linear Functions and Multivariable Local Linearity

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Linear Functions of Two Variables

Definition: $z = f(x, y)$ is a **linear** function of two variables if it satisfies:

$$\begin{aligned} \Delta z &= p\Delta x + q\Delta y, \qquad p \text{ and } q \text{ constant} \\ z - z_0 &= p(x - x_0) + q(y - y_0) \end{aligned}$$

The definition of a linear function of two variables is just an extension of the definition of a linear function of one variable. Many other features of linear functions of one and two variables are also similar:

	One Variable	*Two Variables*
Initial Value Form:	$\Delta y = m\Delta x$ $y - y_0 = m \cdot (x - x_0)$	$\Delta z = p\Delta x + q\Delta y$ $z - z_0 = p \cdot (x - x_0) + q \cdot (y - y_0)$
Intercept Form:	$y(x_0) = y_0$ $y = mx + b$	$z(x_0, y_0) = z_0$ $z = px + qy + r$
Constant Rates of Change:	$dy/dx = m$	$\partial z/\partial x = p, \quad \partial z/\partial y = q$
	graph is a line	surface graph is a plane
Plots:		contour plot with equally spaced levels has (equally spaced) parallel lines provided not both $p = 0$, $q = 0$. slope of lines in contour plot is $\Delta y/\Delta x = -p/q$

Example

1. Write the linear function satisfying $\Delta z = 4\Delta x + 4\Delta y$, $z(2, 2) = 8$, in intercept form.

Local Linearity

Definitions: $z = f(x, y)$ is *locally linear* at $(x, y) = (x_0, y_0)$ if the surface graph approaches a plane as you zoom in on the point $(x_0, y_0, f(x_0, y_0))$. This plane is called the *tangent plane* to the graph at this point. The equation of this plane *approximates* the function near (x_0, y_0). When comparing changes on the the tangent plane with changes on the function graph, we often write increments on the tangent plane as *differentials*.

Local linearity for a function of two variables is an extension of this concept for a function of one variable:

	One Variable $dy = mdx$	*Two Variables* $dz = pdx + qdy$
Equation for Tangent: (Initial Value Form)	$y - y_0 = m \cdot (x - x_0)$ $m = f'(x_0)$ $y_0 = f(x_0)$	$z - z_0 = p \cdot (x - x_0) + q \cdot (y - y_0)$ $p = f_x(x_0, y_0), q = f_y(x_0, y_0)$ $z_0 = f(x_0, y_0)$
Microscope Approximation:	$\Delta y \approx m\Delta x$ $f(x) - y_0 \approx m \cdot (x - x_0)$	$\Delta z \approx p\Delta x + q\Delta y$ $f(x, y) - z_0 \approx p \cdot (x - x_0) + q \cdot (y - y_0)$
Zooming In:	graph approaches tangent line	surface graph approaches tangent plane contour plot approaches contour plot of tangent plane (if tangent plane not horizontal) slope of line tangent to level curve at (x_0, y_0): $dy/dx = -p/q$ $dy/dx = -f_x(x_0, y_0)/f_y(x_0, y_0)$

Example

2. $z = f(x, y) = x^2 + y^2$ is locally linear at $(x_0, y_0) = (2, 2)$. Write the equation, in initial value form, of the plane tangent to the surface graph of f at $(x_0, y_0, z_0) = (2, 2, 8)$. Also write the Microscope Approximation for f at about the point $(x_0, y_0) = (2, 2)$. Compare $f(2.01, 1.95)$ with the approximate value obtained using the Microscope Approximation.